Teaching and Learning Mathematics With Interactive Spreadsheets

Hollylynne Stohl Drier  
North Carolina State University

Learning to teach mathematics at the middle and secondary levels should include many opportunities for teachers to learn how to use technology to better understand mathematics themselves and promote students' learning of mathematical concepts with technology-enabled pedagogy. This article highlights work done in a variety of preservice and in-service mathematics teacher education courses to help teachers use commonly available spreadsheets as an interactive exploratory learning tool. Several examples of teachers' subsequent use of spreadsheets in their own teaching are also discussed.

"Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (National Council of Teachers of Mathematics ([NCTM]), 2000, p.11). This statement is one of six principles described in the NCTM's document, Principles and Standards for School Mathematics. The use of the word essential in the statement has many implications for school mathematics, as well as preservice and in-service mathematics teacher education. Not only are teachers charged with a vision of transforming their teaching and students' learning of mathematics, but teacher educators are challenged with the task of preparing teachers who can utilize technology as an essential tool in developing a deep understanding of mathematics, for themselves and for their students.

Recent trends in teacher education have emphasized the importance of learning with technology rather than learning about technology. This implies that teachers should learn to use a computer as a cognitive tool to enhance student learning of content material (e.g., mathematics, social studies, or science) rather than acquiring isolated skills in basic computing applications (e.g., word processing, database, spreadsheets, or hypermedia) or merely learning a specific programming language (Abramovich & Drier, 1999). Thus to promote the use of technology for students' conceptual development, mathematics teachers should learn how to use widely available software, such as spreadsheets, as a conceptual teaching and learning tool (Abramovich, et al., 1999).

During the past decade, spreadsheets have been used in teacher education and K-12 classrooms to explore a variety of mathematical concepts and to help students use numerical and graphical methods to solve problems (Abramovich, 1995; Abramovich & Nabors, 1998; Clements & Samara, 1997; Dugdale, 1998; Neuwirth, 1995). These uses of spreadsheets allow students to explore alternative solution processes that go beyond symbolic manipulation and can provide students with a deeper understanding of concepts embedded in a problem. One unique use of spreadsheets is the ability to interactively model and simulate mathematical situations. In much the same way scientists use a laboratory to discover and test scientific laws, mathematics teachers can use spreadsheets to create dynamic experiential environments for discovering mathematical relationships. Such activities can facilitate students' engagement with mathematical concepts and their conceptualization of relationships among numerical, graphical, and algebraic representations. The flexibility and power of the Microsoft Excel (1997) spreadsheet software allows teachers to be engaged in meaningful mathematical activities that, in turn, empower them to use spreadsheets as interactive environments in their own teaching.

Promoting Mathematical Exploration with Spreadsheets

Preservice teachers "have little experience with how technology can change the nature and emphasis of the mathematics curriculum" (Gay, 1994, p. 172). The majority of mathematics courses in preservice teachers' backgrounds are taught using traditional instructional methods with little use of computing technologies. In
addition, in-service mathematics teachers are often shuffled through basic training sessions, along with all other content-area teachers, that attempt to teach them how to use basic computing tools (spreadsheet, word processing, and database). Few mathematics teachers walk away from these types of in-service programs with concrete ideas of how to use these tools to help students learn mathematical content.

Teachers can learn to use spreadsheets to create open-ended, interactive teaching and learning environments that can facilitate students’ conceptual development. Interactive spreadsheets can be designed by a teacher to provide students with a laboratory-like environment to investigate a mathematical problem. An interactive spreadsheet can be designed such that the user can perform an action that changes the status of cells, formulas, or graphs in the spreadsheet. When the cells, formulas and graphs are linked, users can observe dynamic changes in multiple representations. By giving preservice and in-service teachers experiences with such powerful spreadsheets, they can, in turn, use their knowledge of mathematics and spreadsheets to design interactive environments for use in their own teaching. This article will highlight a few examples of using interactive spreadsheets with teachers, as well as several examples of interactive spreadsheets created by preservice and in-service teachers for their own classroom use.

Simple Spreadsheet Commands Provide Interactive Environments

In order to introduce teachers to the power of interactive spreadsheets, they explore concepts in probability by using and creating spreadsheets with very simple design elements. For example, the spreadsheet in Figure 1 displays an interface designed to simulate rolling two standard die and analyzing the sum of outcomes for 100 trials. The only formulas used in the design of the spreadsheet are RANDBETWEEN and COUNTIF. The RANDBETWEEN function can only be used after the Analysis Toolpak has been added to Excel. (To add the Analysis Toolpak, under the Tools menu, choose Add-ins..., and then select the Analysis Toolpak option.) The RANDBETWEEN function will randomly choose a number between a given maximum and minimum, inclusive (e.g., RANDBETWEEN(1,6)). The COUNTIF function will search through a given range of cells and count tally if a desired entry is found in a cell (e.g., COUNTIF(K5:K104, "2") will find every instance of the sum 2 within the range of K5:K104). Using those two functions, teachers can use and create environments that quickly, and almost instantaneously, execute a series of random events, tally and display the numerical results, and change a graphical display of the experimental results, all by pressing the F9 key to recalculate the worksheet.

Figure 1. Interactive spreadsheet for experiment with sum of two dice.
Although physical experience with such devices as coins, dice, and spinners is important in developing students' conceptual understanding of probability, electronic simulation can greatly extend and enhance the teaching and learning process. In particular, through electronic simulation, students can develop experiential understandings of such concepts as the law of large numbers and the central limit theorem. To further demonstrate the law of large numbers, teachers could set up several similar worksheets within a single workbook that simulate flipping a coin 10, 100, 500, and 1,000 times. Students can then compare the wide variability in the proportion of heads with a small number of trials to the more stable proportion of heads (near 0.50) in results with a large number of trials.

Linking Cells, Formulas, and Graphs

Another simple way to create an interactive spreadsheet is to use cell references in a formula to calculate values in other cells. For example, by using three cells to represent the coefficients $A$, $B$, and $C$ in the general form of a quadratic function, a table of ordered pairs can be quickly generated (see Figure 2). In Figure 2, the $X$ values in the table range from -10 to 10, while the $Y$ values are calculated using a formula that includes cell references for the coefficients $A$ (B3), $B$ (B4), and $C$ (B5) (e.g., in Figure 2, the formula in cell B8 is $=B6^2+B6^2+B6+5$). As the user changes the values in cell B3, B4, or B5, the spreadsheet recalculates all formulas dependent on those values. Since the values in the table are also shown graphically, changes in these three cells can also be seen almost instantaneously in the graph.

These links between cells, formulas, and graphs allow teachers and students to quickly explore transformations of functions and discover how the coefficients $A$, $B$, and $C$ affect the shape of the resulting graph. In Figure 3, the function $y = x^2$ is represented algebraically, numerically, and graphically. The symmetric shape of the quadratic function can be seen in both the table and graph and can stimulate a discussion of why this function is symmetric about the ordered pair $(0,0)$. By changing only the value of $C$ to 3, the user can numerically observe that all the squared numbers from the function $y = x^2$ have been increased by 3 and view the graphical transformation with $y = x^2 + 3$ (see Figure 3). After changing the value of $C$ several times, students can quickly conclude that the value of $C$ determines the vertical shift in the graph when $B = 0$, and will always be the $Y$-intercept, no matter what values of $A$ and $B$ are used.

The ability to create dynamic multiple representations of a function allows teachers to make the learning

Figure 2. Interactive spreadsheet for exploring transformations of a quadratic function and viewing multiple representations of $y = 1x^2$

![Interactive Spreadsheet for Exploring Transformations of a Quadratic Function](image2)

Figure 3. Multiple representations of $y = 1x^2 + 3$.

![Multiple Representations of $y = 1x^2 + 3$](image3)
of functions interactive, engaging, and exploratory. Although this type of activity can be been done with a graphing calculator, the spreadsheet allows the user to quickly change the values of the coefficients and see a table and graph immediately change. As a follow-up to this exploration, preservice and in-service teachers can create an interactive spreadsheet to explore transformations with quadratic functions using the form \( y = A(x - h)^2 + k \). The teachers then compare how the parameters \( A, B \) and \( C \) from the general form \( y = Ax^2 + Bx + C \), and \( A, h, \) and \( k \) from the transformational form \( y = A(x - h)^2 + k \) affect the graph of the parabola, and discuss mathematical and pedagogical issues of using both forms and having students discover relationships.

Another excellent activity demonstrating the powerful interactive potential of a spreadsheet using basic links between cells, formulas, and graphs is the “sequence generator” from Dugdale (1998). In this activity, Dugdale used basic spreadsheet capabilities to explore patterns numerically and graphically. By changing the defining “rules” for generating a sequence of numbers, students can learn concepts from middle school through precalculus. An adapted form of Dugdale’s activity can be used as a core component of lessons done with preservice and in-service teachers.

**Advanced Interactivity With Form Objects**

Microsoft Excel spreadsheets are especially powerful due to the scroll bars and buttons that allow users to control elements of the spreadsheet or activate a common procedure without repeatedly using spreadsheet functions. This power results from the ability to use object-oriented programming through Visual Basic for Applications within Microsoft Excel. These capabilities are not immediately apparent to novice spreadsheet users. Objects such as scroll bars and buttons can only be accessed by adding additional options to the existing Excel toolbar. To add these options, under the View menu, choose Toolbars..., and then choose Forms (see Figure 4). The Forms toolbar will then appear as a floating box within the spreadsheet.

One of the most useful form objects is the scroll bar (see Figure 4). Once a scroll bar is added to a spreadsheet, it becomes a tactile object that can provide a higher level of interactivity to the environment. For example, the spreadsheet in Figure 5 displays four representations of a rational number (fraction, decimal, percent, and pie graph). By controlling the numerator and denominator with scroll bars, the environment allows a user to change the value of a fraction and dynamically observe the corresponding changes in the decimal, percent, and pie graph. This particular spreadsheet was initially used in an elementary classroom to facilitate an investigation about the relationship between fractions and percents (Drier, 2000). The scroll bars add a quick way for students to explore tasks such as “which is larger 6/21 or 6/22?” To further generalize, students can hold the numerator constant and incrementally increase the denominator. As the denominator increases, students notice that the percent and shaded part of the circle decreases. While using the spreadsheet to find fractions equivalent to 1/2, the tactile motion of

**Figure 4. Inserting the Forms toolbar in Microsoft Excel.**
clicking on the numerator scroll bar once and the denominator scroll bar twice helps students instantiate the constant ratio of 1 to 2 for creating an equivalent class of fractions (e.g., 1/2, 2/4, 3/6, 4/8, 5/10). High school teachers and their students can also use this spreadsheet to model the sequence of fractions 1/9, 2/9, 3/9, ... 9/9 and use the observations to invoke a discussion of whether 0.9999... = 1 is a true equality.

When teachers use the interactive fraction spreadsheet environment to explore several rational number tasks, the powerful use of a scroll bar is readily apparent. Once the forms objects are added to the existing Excel toolbar, a scroll bar can be added to the spreadsheet by clicking on the scroll bar button (Figure 4), and then clicking where the scroll bar is to be placed in the spreadsheet. The scroll bar can be resized and will float over the spreadsheet so it can be moved to the desired position. Once the scroll bar is in place, the user must format the object to define its properties and specify which cell it will control. The menu shown in Figure 6 can be obtained by pointing to the scroll bar object and using a right mouse click (PC) or by holding down the control key and clicking with the mouse (MAC). Once Format Control is selected, the pop-up window in Figure 6 appears. Within this window, the user must define the maximum and minimum whole number values for the scroll bar, as well as the whole number increment when a user clicks on the left or right arrows of the scroll bar. For example, the slider that controls the numerator in Figure 5 has a minimum value of 0, a maximum value of 100 and increments by 1. Notice the box labeled Cell Link (Figure 6) contains the cell reference for C2. This reference provides the essential link for the scroll bar to control a cell's value.

Another advanced interactive spreadsheet can provide an extension to a popular problem used in middle school classes involving finding the maximum area for a rectangular pigpen given a fixed amount of fencing (see Figure 7). This same problem is often used in calculus classes when introducing applications of a derivative. Thus, such a rich problem lends itself well to use in a mathematics education course. Students are first given a handful of square tiles and asked to build all possible rectangular pigpens that have a perimeter of 24 and record the dimensions and resulting area. The maximum area is obtained with a square pigpen. Does a square always provide the maximum area? With only physical manipulatives, students are severely limited in their approach to this problem. The spreadsheet allows students to easily test this conjecture.

The spreadsheet environment in Figure 7 allows users to change parameters of the problem with the scroll bar and dynamically observe the results in numeric, geometric, and graphic forms. Thus, the environment facilitates a search for patterns that can lead to generalizations of the problem. Through repeated experimentation within the environment, teachers recognize that the value of the parameter X that results

---

**Figure 5. Interactive spreadsheet for exploring rational numbers**

![Interactive Spreadsheet](image)

**Note:** The pie graph is created by using cells E2 and E3: E3 contains a hidden formula of 1-E2. Thus, these two cells model both parts of the whole pie.
Figure 6. Formatting a scroll bar to control the values of a cell and the dialog box used to define a scroll bar.

Figure 7. Maximizing area of a rectangle with fixed perimeter.
in the maximum area of the rectangle is one fourth of the perimeter \( P \). To extend this problem through calculus, this relationship is confirmed by using the first derivative of the equation Area = \( X/(P-2X)/2 \) with respect to \( X \).

As seen in the fraction and maximum area spreadsheets, the use of scroll bars to control a cell’s value adds a tactile dimension to a spreadsheet and gives the user quick control of numerical and graphical displays. With careful planning, a teacher can design a spreadsheet and use scroll bars to encourage students to explore “what if” questions that promote mathematical thinking and can give students mathematical power to discover relationships.

**Teachers’ Use of Interactive Spreadsheets**

A major element in teacher education should include planning and implementing lessons that utilize technology as an essential part of the teaching and learning process. Depending on the course, implementation can be done as a peer lesson taught during the course or as part of a field experience associated with the course. In-service teachers enrolled in graduate or professional development courses are often in the ideal situation to actually implement technology-enabled lessons into their current mathematics classrooms. During several preservice and graduate courses taught by the author, teachers have created interactive spreadsheets to help students explore mathematics concepts. The following is a brief account of three teachers’ intentions and a description of why they chose to use an interactive spreadsheet to accomplish their teaching objectives.

**Experimenting With Probability**

Concepts in probability are typically taught from a theoretical perspective with only minimal experimental experiences by students. Amy, then a preservice teacher in a secondary mathematics methods course, wanted to design a lesson for students to use experimental probabilities to closely approximate theoretical ones and to discuss the law of large numbers. She created an interactive spreadsheet that would quickly generate a large amount of random data. The experiment in Amy’s lesson was to draw a random card from a standard deck 2,000 times. The user is able to specify a card (e.g., king of spades), and the spreadsheet tallies the number of
times that card was picked out of the 2,000 trials (Figure 8). In addition, the tallies shown in Figure 9 allow the students to see the similarity in results for picking each of the 52 cards, as well as the 4 suits and the 13 different face values. She used this chart in a peer teaching episode to help other mathematics education students calculate experimental probabilities and to lead them to thinking about the theoretical probability of picking a single card (1/52), a specific suit (1/4), and a specific face value (1/13).

**Studying Distance/Rate/Time Relationships**

The study of equations in algebra often involve applications of the well-known Distance = Rate * Time relationship. However, Betty-Jo, a veteran teacher in the author’s graduate course on teaching and learning algebra, wanted to find a way to allow her students to use a spreadsheet environment to help them tackle some typical distance/rate/time problems. As part of her course project, Betty-Jo created a series of interactive spreadsheets and an accompanying mini-unit she used with her algebra students. One lesson in the unit had students use the spreadsheet in Figure 10 to explore the following problem:

Two buses leave the same station at the same time and travel in opposite directions. After 8 hours, they are 360 km apart. The speed of the faster bus is 3 km/h less than twice that of the slower bus. Find the rate of each bus.

The spreadsheet was created open ended enough to allow students to use it for several different problems. For this particular problem, the user can enter the total distance traveled by both buses in cell E12. The user then needs to create a formula in cell C11 that will calculate the rate of the second bus (RC2) in terms of the rate of the first bus (RC1, cell B5). Then, by setting both time scroll bars to 8 hours, the user can move the scroll bar RC1 and watch the entire spreadsheet recalculate and the stacked bar graph update. When both total distances match, the user has found the required rates for the two buses. Betty-Jo reported that her students were able to find many patterns as they manipulated the scroll bars and seemed to have a better understanding of the relationship between distance, rate, and time. This understanding then helped them conceptualize the paper-and-pencil algebraic solutions to this and other distance/rate/time problems.

**Exploring Projectile Motion**

For a class project in a technology and mathematics education course, Jerril, then a preservice teacher, wanted to use the interactive capabilities of a spreadsheet to help students explore projectile motion and make connections with parametric equations, trigonometry, and quadratic functions. Jerril created the projectile motion environment (Figure 11) to allow students to manipulate various parameters in a parametric equation (initial altitude, initial velocity, and launch angle). With the time scroll bar, students can animate the motion of the object and explore how long it takes for the object to reach its maximum altitude and when the object will hit the ground. By varying the other parameters, the students can explore how each will

---

**Figure 10. Spreadsheet to explore distance/rate/time problems.**

![Spreadsheet screenshot](image)
affect the path of the object and which values will maximize or minimize altitude and horizontal distance. Jerril believed that manipulating different parameters and visualizing the path of the object could make the mathematical equations used to describe projectile motion relevant and meaningful to students. During his peer lesson, he engaged the class in a rich discussion comparing the parameters in the parametric equation to the parameters in trigonometric and quadratic functions.

Summary

Interactive spreadsheets can promote open-ended exploration of mathematical concepts, take advantage of spreadsheet capabilities that allow the learner to extend beyond or significantly enhance what could be done using paper-and-pencil, and give teachers and students an opportunity to discover mathematical concepts in a laboratory-like setting. Creating and using such environments allow teachers to use a spreadsheet to help students make connections between numeric, algebraic, and graphic representations.

With the widespread availability of Microsoft Excel, the use of interactive spreadsheets in mathematics education is a relatively easy way to help teachers transform their classroom into an investigative environment and get students actively engaged in learning. The focus can remain on learning mathematics with technology, rather than merely learning about technology. In addition, the explorations discussed in this article give teachers experience with technology-enabled pedagogy and ways investigative approaches to teaching and learning can help meet the goals of NCTM’s Principles and Standards for School Mathematics (2000).

References


**Editor's Note:** The lessons and spreadsheets discussed in this paper are available at http://curry.edschool.virginia.edu/teacherlink/math/excelactivities.html

Correspondence concerning this article should be addressed to Hollylynne Stohl Drier, Department of Mathematics, Science, and Technology Education, North Carolina State University, 326 Poe Hall, Raleigh, NC 27695.

Electronic mail may be sent via Internet to Hollylynne_drier@ncsu.edu