Fractions Section 3: Fractions as Ratios

Goals

To help students conceive of ratios as fractions, and to be able to see and describe how they see these fractions in pictures they draw to represent the ratios.

Big Ideas

We often think of ratios one of two ways. The first way is in terms of the relationship between two numbers. For example, we might say something like the ratio of girls to boys at the school dance was 3 to 2. Another way of saying this is that for every 3 girls at the dance, there were 2 boys. We can symbolize the ratio of girls to boys by writing 3:2. However, we can also write the ratio of girls to boys as 3/2. What does 3/2 mean in this situation? The 3/2 is 3/2 of what? These questions push us to consider ratios in a second way, namely, as a single number. To help us see what the 3/2 means, consider the picture below, which represents the composition of the population at the school dance:

The entire rectangle represents the total population at the dance. Notice that in order to draw the ratio of girls to boys as 3:2, we needed to divide our population into 5 equal parts. Then 3 parts would be girls and 2 parts would be boys.

Where is the 3/2 in our picture? To be able to see the three halves, we need to be able to see the five equal parts that make up our picture in different ways. Some of the ways we can think about them include

• as fifths of the whole population, because it takes five copies of one of the parts to equal the number of people at the dance;

• as thirds of the girl population, because it takes three copies of one of the parts to equal the number of girls;

• as halves of the boy population, because it takes two copies of one of the parts to equal the number of boys.

Using the last interpretation of what one of the small parts represents, we can see that three of those parts equals the number of girls at the dance, and each part is 1/2 of the number of boys at the dance, so the number of girls at the dance is 3/2 times as many as the number of boys at the dance. In other words, there are 3/2 times as many girls at the dance as boys.
The reason that 3/2 is not immediately obvious from the picture above is that 3/2 comes from a part-part comparison, whereas the fractions we considered in the previous sections were all a result of part-whole comparisons. Naturally, we could also make part-whole comparisons in our picture above, such as concluding that girls represented 3/5 of the whole population. You should be able to see all of these comparisons in the picture.

Notice that 3/2 times as many girls as boys is different from 3/2 times more girls than boys. In order to have 3/2 more girls than boys at the dance, our picture would have to look like the following:

![Diagram of 3/2 more girls than boys]

In this picture, the part of the girls population that is more than the boys population, namely three parts, is 3/2 as many as the number of boys. So the number of girls would be 3/2 times more than the number of boys.