Fraction Section 4: Equivalent Fractions

Goal

Students will be able to simplify fractions by drawing pictures and reasoning about those pictures. They will be able to write written explanations to communicate their reasoning. They will be able to explain what dividing the numerator and denominator by a whole number greater than one accomplishes in their picture, and why a fraction can be simplified if and only if both the numerator and denominator can be divided by a whole number larger than 1.

(Note: We will talk about subdividing fractions to yield equivalent fractions when we discuss common denominators in the section on fraction addition.)

Big Ideas

Equivalent Fractions Using the Measurement Interpretation of Division

Consider the picture below of 20/24:

We can simplify this picture by grouping the 1/24s into larger groups. Below are two different ways of grouping the 1/24s.

In (a), the 1/24s were grouped into groups of 2. We can see twelve equal parts in our whole, so each part is 1/12. Ten of those parts are shaded, so we have 10/12. Because 10/12 and 20/24 represent the same amount of a whole, they are equivalent fractions. This shows that we can simplify the fraction 20/24 to 10/12. By doing so, we not only have a fraction with smaller numbers in the numerator and denominator, but we also have a picture that is easier to read. This suggests that in terms of pictures, simplifying fractions means grouping smaller partitions into larger, equal-sized partitions, such that each new piece is either entirely shaded or entirely unshaded.
Typically we are taught to simplify fractions by dividing the numerator and denominator by the same whole number. To get from 20/24 to 10/12, we would divide 20 by 2 and 24 by 2. How can we see this in our picture? If we use the measurement interpretation of division, this would lead to the questions, How many twos are in 20 and 24? We answer this question in our picture by grouping the 1/24s into groups of 2 so we can see just how many groups of 2 we made. We found that we got 12 groups of 2, the answer to 24 ÷ 2. We were careful, though, to group the shaded 1/24s with other shaded 1/24s (i.e., we didn’t try to form a group of 2 with one shaded and one unshaded 1/24). We created 10 shaded groups of 2, the answer to 20 ÷ 2. Simply put, dividing the numerator and denominator by a particular whole number results in creating larger groups by fusing together that particular number of smaller parts. In the above example, dividing by 2 led to the creation of new, larger groups by repeatedly fusing two 1/24s together.

As we see in (b), 20/24 can be simplified even more. In this picture, we see that 20/24 is equivalent to 5/6. Furthermore, we notice that to get this picture, we grouped the 1/24s into groups of 4. This corresponds to dividing 20 by 4 (to get the number of shaded larger parts) and 24 by 4 (to get the number of larger parts in the whole).

We can see that (b) can no longer be simplified any further. To simplify it, we would have to be able to group the remaining pieces evenly into larger groups, with all the groups consisting of all shaded or all unshaded pieces. This is not possible for 5/6. For example, if we tried to group the six parts into three equal groups, we would end up with a group that would be half shaded. This violates the condition that the simplified fraction consists of parts that are either entirely shaded or entirely unshaded. The symbolic representation, namely (2 1/2)/3, further suggests that trying to simplify the fraction this way leads to a more complex representation than the original 5/6.

Once we understand what dividing the numerator and denominator by a whole number corresponds to in a picture of the fraction, we can make sense of the following statements:

1. **If a whole number greater than 1 divides both the numerator and denominator of a fraction, then the fraction can be simplified:** If a whole number divides the numerator, that means we can make groups of that size from the shaded parts without having any leftover. If the number also divides the denominator, that means we can make groups of that number from the parts that makeup the whole, with none left over. Thus, the fraction can be simplified.

2. **If the only whole number that evenly divides the numerator and denominator is 1, then the fraction is in lowest terms:** If a whole number does not divide the numerator of a fraction, then it is not possible to make groups of that size from the shaded region without having some left over. If a whole number does not divide the denominator, then it is not possible to make groups of that size from the parts that make up the whole without having some left over. Therefore, in order to simplify a fraction, a whole number must divide both the numerator and
denominator. If there are no whole numbers greater than 1 that divide both the numerator and denominator evenly, then the fraction cannot be simplified further.

**Equivalent Fractions Using the Sharing Interpretation of Division**

It is possible to explain the algorithm for simplifying fractions using the sharing interpretation of division, although it may not be as straightforward as the measurement interpretation. To illustrate how the sharing interpretation might be used, consider again the fraction \(\frac{20}{24}\). We can divide the numerator and denominator by 4. Dividing 20 by 4 is equivalent to asking, If I take the 20 shaded \(\frac{1}{24}\)s and divide them evenly into four groups, how many will be in each group? Dividing 24 by 4 asks a similar question, namely, If I take the twenty-four \(\frac{1}{24}\)s that make up a whole and divide them evenly into four groups, how many will be in each group? This leads to the following picture for \(\frac{20}{24}\):

![Diagram of fraction division](image)

From this picture, we see the \(\frac{20}{24}\) is divided into four equal groups, each of which is \(\frac{5}{6}\) shaded. When we use the sharing interpretation, we are not making larger groups made up of all shaded or unshaded parts, like we do when using the measurement interpretation. Instead, we are splitting the shaded and unshaded parts equally among four groups, with the understanding that once we know what fraction of each of these smaller groups is shaded, we will also know what fraction of the whole is shaded. In this case, we see that each of the four equal parts is \(\frac{5}{6}\) shaded. And because the whole is comprised of these four groups, it makes sense that the entire whole is also \(\frac{5}{6}\) shaded.