Math 112 (Calculus I)
Exam 1
Jan 28-30, late day Feb 1

Instructions:

- For questions which require a written answer, show all your work. Full credit will be given only if the necessary work is shown justifying your answer.
- Simplify your answers.
- Calculators are not allowed.
- Should you have need for more space than is allocated to answer a question, use the back of the page the problem is on and indicate this fact.
- Please do not talk about the test with other students until after the last day to take the exam.

For Instructor use only.

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Part I: Multiple Choice  Mark the correct answer on the bubble sheet provided.

1. The domain of \( f(x) = \sin(\sqrt{x}) + 1/x \) is:
   a) \((-\infty, \infty)\)          b) \([-1, 1]\)          c) \((-1, 1)\)
   d) \([0, \infty)\)          e) \((0, \infty)\)

2. Which of the following statements is true?
   a) The domain of the function \( y = |x^2 - 1| \) is \([0, \infty)\).
   b) The graph of \( y = f(-x) \) is a reflection of the graph of \( y = f(x) \) about the x-axis.
   c) If \( c > 1 \) then \( y = f(cx) \) compresses the graph of \( y = f(x) \) horizontally.
   d) The function \( y = -e^{x^2} \) is odd.
   e) The domain of \( y = e^x \) is all real numbers and the range is \([0, \infty)\).

3. If \( 5e^{x/2} = 4 \), solve for \( x \).
   a) \( 8/5 \)          b) \( 2 \ln 4 + \ln 5 \)          c) \( \ln(16/25) \)
   d) \( 2e^{4/5} \)          e) \( 2 \ln(5/4) \)          f) \( \ln 4 - \ln 25 \)

4. Which of the following lists all of the vertical asymptotes of the function \( f(x) = \frac{x + 1}{(x^2 - 1)(x - 3)} \)?
   a) \( x = 3 \)          b) \( x = -1 \)          c) \( x = 1 \)
   d) \( x = 3, x = -1 \)          e) \( x = 3, x = 1 \)          f) \( x = 3, x = -1, x = 1 \)

5. Let \( f(x) = \frac{x^2 - 81}{|x - 9|} \). Find the limits \( \lim_{x \to 9^+} f(x) \) and \( \lim_{x \to 9^-} f(x) \)
   a) 18 and 9          b) 18 and -18          c) both 18
   d) 18 and -9          e) both 9          f) 9 and -9
   g) both undefined          h) \(-\infty \) and \( \infty \)          i) \( \infty \) and \(-\infty \)
6. Evaluate the limit \( \lim_{x\to 2} \frac{2x - 4}{\sqrt{2x} - 2} \)
   
   a) \( \infty \)  
   b) \( -\infty \)  
   c) 0  
   d) 1  
   e) 2  
   f) 4

7. Suppose you know that \( \lim_{x\to -3} (2x + 7) = 1 \). Which of the following values for \( \delta \) would guarantee that
   \[ |(2x + 7) - 1| < 0.01 \] if \( 0 < |x + 3| < \delta \)?

   Select the largest correct answer below.
   
   a) 0.001  
   b) 0.005  
   c) 0.009  
   d) 0.012  
   e) 0.021  
   f) None of the above.

8. Find a value of \( c \) for which the function below is continuous everywhere.

   \[ f(x) = \begin{cases} 
   cx^2 + 2x + 2 & x < 1 \\
   x^2 - 3x - c & x \geq 1 
   \end{cases} \]

   a) 0  
   b) 1  
   c) \(-8\)  
   d) 5  
   e) \(-3\)  
   f) None of the above.
Part II: Show all work.

9. (7 points) Let \( f(x) = 2x^2 - 5x + 1 \) and \( h \neq 0 \). Evaluate \( \frac{f(a + h) - f(a)}{h} \).

10. (9 points) Sketch the graph of \( y = 2 - \sqrt{x+4} \). State the domain and range.

\[
\begin{array}{c}
  y \\
  \hline \\
  x
\end{array}
\]

11. (7 points) Express the function \( F(x) = 4 - e^{x^3} \) in the form \( f \circ g \circ h \) for appropriate functions \( f, g \) and \( h \).
12. (7 points) Simplify the following as an algebraic function.

\[ \cos(\tan^{-1} \frac{x}{3}) \]

13. (9 points) Find the inverse of the following function.

\[ y = \frac{x - 1}{3x + 2} \]

14. (7 points) Let \( f(x) = \begin{cases} 
1 & \text{if } x < -1 \\
\frac{1}{x+1} & \text{if } x = -1 \\
\frac{1}{x^2} & \text{if } -1 < x < 3 \\
11 & \text{if } x = 3 \\
9 & \text{if } 3 < x 
\end{cases} \). Determine whether the following limits exist, and if so find their value:

\[ \lim_{x \to -1^-} f(x), \lim_{x \to -1^+} f(x), \lim_{x \to -1} f(x), \lim_{x \to 3^-} f(x), \lim_{x \to 3^+} f(x), \lim_{x \to 3} f(x), \]
15. (7 points) Suppose $\lim_{x \to a} f(x) = 0$, $\lim_{x \to a} g(x) = -2$, $\lim_{x \to a} h(x) = -3$. Find

$$\lim_{x \to a} \cos(2f(x) - g(x) + 3h(x))$$

and justify each step by indicating the appropriate limit law(s).

16. (7 points) Show that $\lim_{x \to 0} |x| \cos(\pi/x) = 0$

17. (7 points) Show that $\ln x = 3 - 2x$ has at least one real solution.
18. (9 points) Prove the following statement using the epsilon, delta definition of limit.

$$\lim_\limits{x \to 4} (3x - 10) = 2.$$