Using the Internet to Illuminate NCTM’s Principles and Standards for School Mathematics

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The National Council of Teachers of Mathematics (NCTM) Illuminations project was launched in 1998 to support NCTM’s Principles and Standards for School Mathematics, which was then still under development. The mission of Illuminations is to improve the teaching and learning of mathematics for all students by providing high-quality Internet resources that “illuminate” Principles and Standards.

Overview of Illuminations

The first author of this article became the director of the project in 1999, and the other authors joined shortly thereafter, forming the development team that conceptualized the project and built the Illuminations Web site (see fig. 15.1). From 1999 until 2003, when our participation on the project ended, the Illuminations Web site grew to become one of the leading school mathematics Web sites on the Internet. Visits to the site increased from only a few hundred in September 1999 to about 331,000 visits in October 2003. Each visit is one person staying on the site for an average of about eight minutes; the number of “hits” (clicks on any link on the site) in October 2003 was more than 9.75 million (WebTrends report, October 2003). Note that since this article was written, the site and its content have been redesigned and will continue to evolve, so that some of the details discussed in this paper may be different from what is on the site today.

Editor’s note: The CD accompanying this yearbook contains an electronic version of this article, complete with hyperlinked URLs.
Illuminations is a partner in the MarcoPolo Internet Content for the Classroom program (www.marcopolo-education.org), funded by the MarcoPolo Education Foundation. The partners in this program are NCTM, the American Association for the Advancement of Science, the National Council of Teachers of English, the International Reading Association, the National Council on Economic Education, the National Endowment for the Humanities, the Kennedy Center for the Performing Arts, and the National Geographic Society. Additional funding for the project has been generously provided by the GE Fund, the philanthropic foundation of the General Electric Company.

The mission of Illuminations has been carried out by pursuing four main development strands:

1. Online, interactive, multimedia resources (primarily using applets and videos)
2. Internet-based lesson plans
3. Reviewed and categorized external Web resources
4. A Web design framework that organizes and presents the content in such a way that the design itself helps illuminate *Principles and Standards* and makes all content as usable and accessible as possible.
The Internet resources on Illuminations serve many purposes for many audiences, by furnishing professional development and teaching resources for teachers, offering rich classroom materials for students, communicating the vision of Principles and Standards for all users, and providing outreach and an Internet portal for NCTM. Primarily, though, these resources are for teachers, to help them understand and enact the vision of Principles and Standards and to teach mathematics more effectively to their students.

In this article we will discuss each of the four development goals listed above, and we will present informal case studies of how Illuminations has been used for professional development with preservice and in-service teachers. But first we discuss some general principles that guide the development work.

**Guiding Principles for Site Development**

The overarching principle guiding the design and content development is the basic goal to illuminate Principles and Standards. Moreover, other guiding principles are drawn directly from this document. For example, we have developed the Web site on the basis of the recommendation that “[s]tudents’ understanding of mathematical ideas can be built throughout their school years if they actively engage in tasks and experiences designed to deepen and connect their knowledge” (NCTM 2000, p. 21). Likewise, to truly understand the vision put forth in Principles and Standards, one must actively engage in doing mathematics in a manner that is reflective of that vision. Thus, we have focused on creating peer-reviewed resources that engage teachers, students, and all visitors to the site in doing and thinking about mathematics.

As all teachers know, a principal factor in successful teaching and learning is student motivation. Related to this idea, a research study in the *Journal of Computers in Mathematics and Science Teaching* states, “In summary, we observed that the motivational aspect of learning with the Web likely results from a number of factors, including … interactivity, variable entry, data base fascination, and multimedia” (Moor and Zazkis 2000, p. 101). We have developed Illuminations with these factors in mind. For example, interactivity and variable entries are built into the Illuminations interactive mathematics investigations, database fascination is exploited in the carefully organized collection of reviewed external Web resources, and multimedia experiences are prevalent throughout the site.

Central to the development of the Illuminations resources is our conceptualization of the Internet as a global, online, interactive, interconnected, multimedia platform on which one can build and deliver powerful educational and professional development experiences. How can effective Web-based learning environments be built on this platform? An article in the *Journal of*
Research on Computing in Education reports that “the pedagogical approaches … for the development of valuable learning environments are still far from being implemented in most educational Web sites” (Mioduser et al. 2000). In this study, the researchers analyzed the pedagogical characteristics of 436 Web sites that they classified as “web-based learning environments.” They found, for example, that only 28.2 percent of the sites include inquiry-based activities, only 32.6 percent focus on analysis and inference processes, barely 5 percent focus on problem solving and decision making, and just 2.8 percent of the sites support any form of collaborative learning. The Illuminations Web site has been designed to offer resources that embody all these effective learning processes, as well as other recommendations in Principles and Standards.

In fact, the Principles themselves—Equity, Curriculum, Teaching, Learning, Assessment, and Technology—have helped shape Illuminations. For example, much of the site is constructed around mathematics investigations that are intended to engage all students in learning important mathematics. Many of the investigations and lesson plans are threaded together to present local examples of developing a coherent curriculum of connected concepts and strands of mathematics. Resources such as classroom video vignettes and guiding questions for teaching and assessing are presented so that the investigations and lessons offer opportunities for teachers to reflect on the practice of teaching. Technology, of course, is integrated throughout the site and serves as a backbone to the project. These points will be elaborated and illustrated below, beginning with the following discussion of the different types of Illuminations resources.

Illuminations Online, Interactive, Multimedia Resources

The Illuminations Web site contains online, interactive tools for teaching, learning, and professional development in mathematics. The different types of online tools include (a) general purpose tools such as a spreadsheet, a grapher, and a shape manipulator; (b) content-specific tools, such as applets that facilitate the exploration of linear regression or fractions; (c) context-specific tools, such as applets that simulate light intensity or a game of pool; and (d) professional reflection tools, such as online video vignettes of students learning and teachers teaching mathematics. All these tools are integrated into mathematical or professional development activities that can be used to help achieve the vision of Principles and Standards. These online activities furnish examples of effective uses of the tools. The examples might be used directly
in classrooms, workshops, or individually; they might be modified in many ways; or they might stimulate ideas for entirely new activities using the same or different tools.

For example, consider an interactive math applet that is tied to the specific context of a game of pool (illuminations.nctm.org/index_o.aspx?id=125). The pool game applet tool is embedded into an online mathematical investigation in a way that is in-line and just-in-time. That is, the applet is in place and ready to click and use when you need it. In the investigation, middle school students are asked to determine the ending corner, number of hits, and length of the path when a cue ball is hit at a 45° angle from one corner of a pool table.

This task supplies a rich context for students to further their understanding of ratio, proportion, greatest common factor, least common multiple, and symmetry. Although the investigation can be accomplished without technology, the applet significantly enhances teaching and learning by reducing the physical difficulties of carrying out the investigation, reducing the amount of time needed to gather necessary data, offering an easy method for testing conjectures and getting immediate feedback, increasing student ownership of the problem, and connecting a concrete problem to an abstracted grid representation.

Depending on the purpose and resources available, the applet-based pool-game activity can be used in the classroom in many ways. For example, this could be (1) a whole-class activity where a single computer is used to demon-
strate and verify students’ ideas, (2) a small-group activity in a computer lab, (3) an out-of-class activity in which students experiment and gather data that are discussed in subsequent classes, or (4) an end-of-unit project.

Other Illuminations tools are used to create online, interactive, multimedia professional development activities. These activities are currently built around video vignettes of teachers teaching and students learning mathematics. For example, the professional development activity Teaching, Learning, and Communicating about Fractions (illuminations.nctm.org/reflections/3–5/fractions/index.html) uses classroom videos of students playing a fraction game. The fraction game itself can be played online using two different applets in a one-player (illuminations.nctm.org/tools/fraction/fraction.asp) or two-player version (standards.nctm.org/document/eexamples/chap5/5.1/index.htm). The classroom video vignettes show teachers modeling good communication skills as they encourage their students to communicate mathematically and think carefully about the fraction concepts. The videos and associated activities help teachers develop their ability to pose questions that elicit, extend, and challenge students’ thinking, which is an essential part of creating a classroom environment in which intellectual risks, sense making, and deep understanding are expected.

As an example that uses online applets and videos together with off-line mathematical tools, consider Shedding Light on the Subject: Function Models of Light Decay (illuminations.nctm.org/index_o.aspx?id=137). This investigation includes an applet-tool simulating light intensity during an underwater dive, an off-line CBL experiment, an applet or use of calculators for analyzing data, video clips of students engaged in the investigation, teacher notes, and solutions. This activity also offers an example of how technology can influence what mathematics we teach, in this example more statistics and discrete mathematics, a functions approach to algebra, and more real-world mathematical modeling. See figure 15.2.

**Illuminations Internet-Based Lesson Plans**

Now we discuss the second major type of content found on Illuminations: Internet-based lesson plans. Illuminations lessons show how the Internet can be used for Standards-based mathematics lessons. For example, Internet links can furnish real-world data that can be analyzed or used to develop mathematical concepts. Other links may supply detailed information about areas in which mathematics is applied. Some links offer tools that can be used to graph, visualize, or compute. In all examples, the Internet links are used to enhance students’ learning and promote more effective teaching.

A primary message that we endeavored to convey and build into the les-
sons is that “in planning individual lessons, teachers should strive to organize
the mathematics so that fundamental ideas form an integrated whole” (NCTM
2000, p. 15). Toward that end, Illuminations lesson plans are in fact most
often unit plans composed of five to seven sequenced lessons that develop
substantial mathematical ideas across longer time periods. Because the plans
have the complementary purposes of providing teaching resources, offering
professional development for teachers, and communicating the vision of
Principles and Standards, they are carefully crafted to present important
mathematical ideas and effective instructional practices.

All lessons include questions to guide the development of mathematical
understanding, suggestions for ongoing assessment experiences and proce-
dures, a variety of instructional strategies, and questions to guide teachers’
reflection. The unit plans demonstrate the value of technology as a tool for
advancing and enhancing students’ mathematical knowledge and their ability
to use mathematics.

Notably, the unit plans are designed to accomplish the following:

• Illuminate and communicate the new vision of school mathematics presented
  in NCTM’s Principles and Standards for School Mathematics
• Ensure access to *Principles and Standards* for many teachers at many levels of knowledge and experience

• Be examples of how to use the Internet in the classroom, taking advantage of the interconnected, interactive, multimedia nature of the Internet

• Offer opportunities for teachers at different levels of knowledge and experience to experience *Standards*-based teaching

• Furnish examples of what’s important in a *Standards*-based lesson, including guiding questions, reflection activities, and sound and significant mathematics

• Facilitate effective classroom practice

For example, a unit plan for prekindergarten to grade 2 entitled “How Many More Fish?” (illuminations.nctm.org/index_o.aspx?id=51) consists of seven sequenced lessons that engage students in actively investigating five meanings and representations for the operation of subtraction: counting, sets, number line, balance, and the inverse of addition. See figure 15.3.

**Illuminations Reviewed External Web Resources**

The Internet is certainly valuable as a vast collection of information and resources. But this value quickly turns problematic as the quantity of information grows along with the time required to find what one is looking for. The Illuminations Web site particularly addresses this opportunity and problem through its collection of reviewed external Web resources (illuminations.nctm.org/swr/index.asp). In brief, this collection of resources gives *Standards*-focused access to the vast Internet virtual library.

These reviewed resources have four principal characteristics that make them useful for *Standards*-based mathematics education. First of all, the resources are organized according to the Standards and Expectations of *Principles and Standards*. Second, they are selected on the basis of a rigorous review by mathematics educators and mathematicians. Third, a written review of each resource is presented, which describes the resource and why it is useful for effective teaching and learning of mathematics and lists possible caveats that should be considered. And fourth, the resources are chosen at a small grain-size, often just one page on a larger Web site, focused on a particular topic and grade, so that they provide high-quality Web resources to help teachers teach specific topics at specific grade levels.

An objection that might be raised to this collection of resources is that it promotes a “bits and pieces” approach to the curriculum. Our view is that the resources we collect should be regarded as sources of information as the teacher weaves a coherent, focused curriculum. They should not be used as a curriculum in itself.
Here is an example of how this collection could be used. Suppose a first-grade teacher will be starting on a unit on place value.

- **Step 1.** She selects “Number and Operations” for “Pre-K–2” from the navigation matrix in which the reviewed external resources are organized, as shown in figure 15.4.

- **Step 2.** She is given a list of descriptions of the available resources, which she then scans to see what might be appropriate. A partial list is shown in figure 15.5.

- **Step 3.** She clicks on the fourth item to learn more. She is given the following review (fig. 15.6) of the site, which suggests it might meet her needs, so she visits the site to see it for herself. The review also points out that Java is needed, so she checks with the resource center to be sure the computers in the lab have Java.

This example illustrates the most global search of the external Web resources collection. This procedure is the front end and navigation that users first encounter, so that they get a friendly, holistic vision of the collection. However, one might want to do a more targeted search. One could use a generic search engine, but instead we have developed what is essentially a *Standards*-based search engine. This system is an example of “contextualized
navigation,” which is discussed in more detail in the Web design section below. The advantage of this system is that it categorizes and searches the collection in an efficient manner that is directly tied to and helps illuminate *Principles and Standards*. Here is how it works for the same first-grade situation we considered above:

- **Step 1.** Start with the following search entry: *place value*
- **Step 2.** This step generates the following “key phrases” related to place value, which are taken directly from the Standards and Expectations of *Principles and Standards*:

![Fig. 15.4](image1)

![Fig. 15.5](image2)
Selected Web Resource: Base Ten Blocks

Resource Review
This Web site consists of a Java applet, where one can select any combination of three different block sizes (representing a unit, 10 units, and 100 units) and drag them into the working panel. Students can then move, rotate, break, and glue the blocks to explore base 10 place value; older students in this grade band can explore multi-digit addition and subtraction. You can begin with the instructions or jump directly to the applet.

While virtual manipulatives may never replace the use of physical materials, there are some advantages. First, students can actually break apart the virtual blocks to decompose them into smaller blocks or glue collections of 10 blocks together to make larger blocks. With physical blocks, one has to "trade" a collection of blocks for another block (or v.v.). Second, the system constrains the giving and breaking of blocks, so that incorrect regroupings cannot occur. The feedback may help students stay on track as they begin to explore place value and combining multi-digit numbers. This site could be appropriate either for individual student use (if appropriate technology, tasks, and guidance are available) or as a tool for teachers to lead a full-class discussion.

Fig. 15.6

- Base-ten blocks
- Comparing numeration systems
- Extending base-ten understandings
- Extending place value
- Introducing the base-ten numeration system
- Introducing place value
- Understanding place value

Step 3. Choose the last of these, “Understanding place value.” This relates to the Standards and Expectations as follows:

- Standard: Number and Operations (Pre-K–2)
- Goal: Understand numbers, ways of representing numbers, relationships among numbers, and number systems
- Expectation: Use multiple models to develop initial understandings of place value and the base-ten number system

Step 4. The “Understanding place value” key phrase generates the following list of Illuminations reviewed external Web resources that are assigned to that key phrase:

- A Fictional History of Place Value
- Base Ten Blocks
- Counting the Rice
• Introduction to Place Value with Corn
• Place Value
• Scribble Square

• Step 5. Of these, the second, Base Ten Blocks, is the same resource used in the
previous example, and the user gets the same review shown above.

Using Illuminations with Preservice Teachers

We will now present two brief data-based illustrations of how
Illuminations has been used for professional development. In this and the next
section we describe how Illuminations has been used with preservice and in-
service teachers, respectively.

Principles and Standards recommends that 2-D and 3-D spatial visualiza-
tion and reasoning are core skills that all students should develop. For exam-
ple, students in grades 3–5 “should become experienced in using a variety of
representations for three-dimensional shapes” (NCTM 2000, p. 169), such as
isometric drawings, a set of views (e.g., top, front, right), and building plans.
Spatial visualization has been defined as the “comprehension and perform-
ance of imagined movements of objects in two- and three-dimensional space”
(Clements and Battista 1992, p. 444).

Isometric drawings have long been recognized as being difficult for both
students and preservice teachers. Some of the sources of this difficulty
include the inherent use of perspective, the projective nature of isometric
drawings, which, for example, results in hidden lines and cubes and multiple
realizations of any drawing, unfamiliar orientation, that is, the lack of “stan-
dard” x-, y-, and z-axes, and translation from isometric drawings to other rep-
resentations, such as top-right-front views.

To help achieve the recommendation from Principles and Standards and
overcome typical difficulties with isometric drawings, an isometric drawing
applet (illuminations.nctm.org/tools/isometric/isometric.asp) was created in
which students could explore the complexities of isometric drawings. The
applet we created allows students to create dynamic drawings of three-dimen-
sional objects on an isometric-dot grid (see fig. 15.7). Using this applet, the
teacher can create isometric drawings with cubes, faces, or edges. Students
can rotate, shift, color, decompose, and view them in 3D or 2D. In addition to
the main isometric view, students can click the “eye” icon in the top menu bar
to see two other views—the front-right-top views (also called “shadow views”) that consist of three particular 2-D projections, and the mat plan or
“building plan” view (see fig. 15.8).
Also, objects built in the main window can be rotated in a new window (see fig. 15.9). This new rotated view is dynamically linked to the isometric drawing so that changes made in the object are automatically reflected in the new window. This feature is helpful when investigating problems associated with hidden cubes and other issues of projection.

For example, consider “impossible figures”—Escher-like drawings that can be interpreted as being impossible to build (see fig. 15.10). By rotating, students can get a new “illuminating” perspective on these figures.
Several features of the applet have the potential to improve students’ spatial skills and understanding of 2-D representations. Students can use the applet to create and reason about many more drawings and views than if the drawings are done by hand on paper. This is a common benefit of using technology and should facilitate the development of skills and understanding. However, we believed that the applet must accomplish more than just this potential benefit. Thus, in addition, the applet is designed so that objects can be drawn using cubes rather than using line segments as with paper draw-
Fig. 15.8. Front-right-top views (a) and mat plan view (b) included in the applet

Fig. 15.9. Screen shot of a window for rotating a constructed object
ings, thereby promoting a direct connection to the physical manipulation of concrete blocks. Also, the cubes can be moved without the limitations of physical space. Finally, multiple representations and views are furnished, including isometric views, front-right-top views, mat plans, and rotations through different axes. Moreover, these representations are all dynamically linked, so that whenever the user changes one view, the effect on the other views is immediately shown. The educational value of multiple linked representations has often been discussed and documented (e.g., in the Representation Standard in *Principles and Standards*). The use of technology, and this applet in particular, greatly facilitates creating and translating among multiple representations.

To help achieve the potential educational benefits of all these features of the applet, we created a set of online instructional tasks that engage students in using the applet to develop their spatial visualization skills. That is, we created an online unit by embedding the applet in rich curriculum materials. Rather than develop materials from scratch, and to maximize the usefulness of these materials, we have linked our Illuminations development work to prominent curriculum development projects with which we are collaborating. For example, the Connected Mathematics Project, which is a middle school curriculum development project funded by the National Science Foundation and designated Exemplary by the U.S. Department of Education, includes a

![Fig. 15.10. An “Escher-like” figure, one in which the “natural” interpretation contains some contradiction; imagine walking along the top of the figure.](image-url)
A geometry unit entitled “Ruins of Montarek” (Lappan et al. 2002). In this unit, students explore two-dimensional views of three-dimensional objects, including front-right-top views, isometric views, and mat plans. However, noticeably absent from these materials is the use of technology. We worked with the authors of this widely used middle school curriculum to develop an online applet-based enhancement of this unit.

We have used this curriculum-embedded applet as part of a course for pre-service teachers. Sixty undergraduates (two sections) worked through this unit as part of a geometry course for future elementary school teachers at a large state university in the Midwest. For example, students used the applet to help them draw an isometric representation. Toward this end, students could be seen holding their drawings to the screen to make comparisons. Of course some students could record their results without needing to refer back to the screen; they sometimes referred to this process as drawing “from memory” (see fig. 15.11).

Students were evaluated on a written test with five targeted categories: Correct Drawings, Shading, Build from Front-Right-Top views, Visualization, and Awareness. The responses on the written tests were evaluated as positive, negative, or not informative. A student was considered “proficient” in a category if 80 percent of the items related to that category were evaluated as positive. Figure 15.12 shows the percent of students reaching proficiency in the five targeted categories.

This discussion presents just a brief preliminary evaluation of the effectiveness of the applet-based approach to teaching and learning about isometric drawing. Our primary evaluative goal was to determine if learning was occurring in the intended categories. Overall, as figure 15.12 suggests, students’ ability did improve in all five categories.

Using Illuminations with In-Service Teachers

In this section, we present an example of how Illuminations has been used with in-service teachers. Illuminations was used to furnish professional development for middle grades teachers participating in an Eisenhower project at a large southern state university. Relying on the vision set forth in Principles and Standards for School Mathematics, this project proposes to support teachers’ professional development, including their knowledge of mathematics, their ability to assess learning, their leadership among colleagues, and their skill in integrating technology.

During the summer of 2001, three of the authors of this article facilitated a three-day workshop for thirty local middle school mathematics teachers. The workshop used Illuminations resources to provide professional development
Fig. 15.11. Two students’ methods of recording the results of an exploration—one relying on the screen and the other recording “from memory”

Fig. 15.12. Percent of students reaching a level of proficiency by category
related to the *Standards* documents for grades 6 through 8. Later in the summer, participants developed plans for implementing ideas from the workshop into their classes. During the academic year, these plans were carried out with support and monitoring from the project staff. The teachers and project staff recognized the potential of Illuminations applets and lesson plans for developing mathematical content knowledge, for improving the design and delivery of lessons, and for increasing students' interest and achievement in mathematics.

The dynamic, visual nature of the applets gave teachers a “picture” of mathematical concepts. For example, the slider on the fraction applet shows proportional relationships among fractions. This feature caused the teachers to rethink ways to help students (and themselves) understand proportionality and the part-whole relationship of fractions. Guiding questions, which are an important feature of Illuminations lessons, helped teachers consider the types of questions that would help focus students’ attention on making sense of the mathematical concepts and the skills and applications associated with those concepts. When reviewing the format and sequence of an Illuminations lesson, the teachers recognized that a thoughtful presentation of the lesson in a logical sequence helped students recognize and make connections that build understanding. The idea of including assessment as a regular part of the instructional process was new to many teachers, yet by using the Illuminations lessons, they saw the value for students—for example, to help students build ownership of their own learning by explicitly perceiving their growth toward understanding the mathematical learning targets. The teachers also saw the value of monitoring the learning of their students throughout the unit, since this allowed them to change instructional activities more effectively, plan grouping strategies so that students could learn more effectively from one another, and know which students needed remediation at each step of the way. And finally, through the reflection activities that are part of many Illuminations resources, the teachers began to see the value of, and engage in, the process of regular reflection on their teaching in regard to its impact on their students’ learning.

**Conclusion**

In this article, we have described the main types of Illuminations content, discussed the principles guiding the development of the content and the Web site design, and given examples of how Illuminations has been used for professional development. The Illuminations Web site continues to change and grow, but it maintains its focus on illuminating the recommendations for improving the teaching and learning of all students put forth in NCTM’s *Principles and Standards for School Mathematics.*
REFERENCES


