3. Statement: If P is any point, then there are at least two distinct lines \( l \) and \( m \) such that P lies on both \( l \) and \( m \).
   Proof: Let P be a point. Let A, B, C be three distinct non-collinear points (Axiom 3). Let \( l, m, n \) be the lines on which the points PA, PB, and PC lie, respectively (Axiom 1).
   Now I am going to show that not all three of these lines are the same, which will mean that at least two of these lines are distinct, thus proving the statement. Suppose that \( l = m = n \) (RRA). Then since A lies on \( l \), B lies on \( m \), and C lies on \( n \), this would imply that all of the points lie on the same line. But these points were chosen so that they are not non-collinear, which implies a contradiction. Thus, at least two of these lines must be distinct, which gives us two distinct lines on which P lies.

4. Statement: If \( l \) is any line, then there exists lines \( m \) and \( n \) such that \( l, m, \) and \( n \) are distinct and both \( m \) and \( n \) intersect \( l \).
   Proof: Let P and Q lie on \( l \) (Axiom 2). By theorem 2.6.3, there is a point R that does not lie \( l \). Let \( m \) and \( n \) be the lines on which PR and QR lie on, respectively. The line \( l \) is distinct from these two lines, because if \( l \) is equal to either of these lines, it would imply that R lies on \( l \), a contradiction. Likewise, if \( m = n \), then \( m \) would contain P, Q, R, and since there is only one distinct line on which P and Q lie, then \( l = m \), which we have already shown is not possible. Thus, \( l, m, \) and \( n \) are distinct lines, and \( m \) and \( n \) intersect \( l \).

5. Statement: If P is any point, then there exists at least one line \( l \) such that P does not lie on \( l \).
   Proof: Let P be a point, and A, B, C distinct points that are non-collinear. Let \( l, m, \) and \( n \) be the lines on which AB, AC, and BC lie, respectively
   Now I am going to show that all three of these lines are distinct. Suppose that \( l = m \) (RRA). Then since A, B lie on \( l \) and B, C lie on \( m \), this would imply that A, B, C are collinear. But these points were chosen so that they are not non-collinear, which implies a contradiction. Using a similar argument, we could show that \( l \neq n \) and \( m \neq n \). So the three lines must be distinct.
   Now I am going to show that P can’t lie on all three lines. Suppose that P lies on \( l, m, \) and \( n \) (RRA). Then since APB lie on \( l \) and BPC lie on \( n \), we have two lines on which both B and P lie. But only one such line can exist (Axiom 1), which implies \( l = n \). This contradicts the previous result. So P must not lie on one of the lines. Without loss of generality, let \( l \) be this line. This proves our result.

7. Statement: If P is any point, then there exist points Q and R such that P, Q and R are non-collinear.
   Proof: Let P be a point. Let Q be another point (Axiom 3), and \( l \) be the line on which both P and Q lie (Axiom 1). Suppose that there is no other point R such that R does not lie on \( l \) (RRA). Then all other points lie on \( l \). But this implies that all points are collinear, which contradicts Axiom 3. Thus, there must be another point R that does not lie on \( l \). Since \( l \) is the only line on which P and Q can both lie (Axiom 1), this implies that no other line can contain P, Q, and R. So P, Q, and R must be non-collinear.