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HELEN M. DOERR and ROXANA ZANGOR

CREATING MEANING FOR AND WITH THE GRAPHING CALCULATOR

ABSTRACT. In this study, we seek to describe how the meaning of a tool was co-constructed by the students and their teacher and how the students used the tool to construct mathematical meaning out of particular tasks. We report the results of a qualitative, classroom-based study that examined (1) the role, knowledge and beliefs of a pre-calculus teacher, (2) how students used graphing calculators in support of their learning of mathematics, (3) the relationship and interactions between the teacher's role, knowledge and beliefs and the students' use of the graphing calculator in learning mathematics, and (4) some constraints of the graphing calculator technology that emerged within the classroom practice. We found five patterns and modes of graphing calculator tool use emerged in this practice: computational tool, transformational tool, data collection and analysis tool, visualizing tool, and checking tool. The results of this study suggest that nature of the mathematical tasks and the role, knowledge and beliefs of the teacher influenced the emergence of such rich usage of the graphing calculator. We also found that the use of the calculator as a personal device can inhibit communication in a small group setting, while its use as a shared device supported mathematical learning in the whole class setting.

KEY WORDS: graphing calculator, tools, pre-calculus, technology, teacher knowledge

1. INTRODUCTION

Functions and graphs have been the focus of numerous research studies over the past decade. The study of students' understanding of the concept of function, and their abilities to create and interpret graphical representations, was given strong impetus by the advent of computers and their ready availability in some classrooms. This led to many computer-based studies that analyzed students' reasoning with and about linked, dynamic multiple representations of functions (e.g., Confrey and Doerr, 1996; Leinhardt, Zaslavsky and Stein, 1990; Moschkovich, Schoenfeld and Arcavi, 1993; Yerushalmy, 1991). Yet despite the limitations of the graphing calculator when compared to a full-screen computer program, the calculator's low cost, portability and ease of use have resulted in its widespread use for teaching about functions and graphs in secondary schools in the United States.

Over the past decade, secondary mathematics teachers have moved to adapt the graphing calculator into their practice. The National Council of Teachers of Mathematics's (NCTM) curriculum standards (NCTM, 1989) recommend using the graphing calculators to provide students with new approaches, such as the use of multiple representations, to the investigation of mathematical ideas. While it might appear that practice has moved independently of and more quickly than research, there is in fact no shortage of research studies on the use of the graphing calculator. See Dunham and Dick (1994), Wilson and Krapfl (1994), and Penglase and Arnold (1996) for extensive reviews. Many, if not most, graphing calculator studies are quasi-experimental in design and seek to answer the question of whether or not graphing calculators are effective in achieving certain instructional objectives, which are often left unchanged from traditional paper-and-pencil approaches (Adams, 1997; Quesada and Maxwell, 1994; Ruthven, 1990; Shoaf-Grubbs, 1994). Many such studies compare the use of the graphing calculator to the use of paper and pencil on the same set of tasks, giving only limited insight into how and why students use graphing calculators in the instructional context. One such insight is provided in the work of Ruthven (1990), who found that students developed three distinct approaches to symbolizing a graph: an analytic-construction approach that exploits mathematical knowledge, a graphic-trial approach that compares successive expression graphs with the given graph, and a numeric-trial approach guided by the coordinates of the given graph.

The relationship between teachers' knowledge and pedagogical strategies and their use of the graphing calculator is largely unexamined. Many studies (Drijvers and Doorman, 1996; Hollar and Norwood, 1999; Porzio, 1997) do not report or describe the role of the teacher in the classroom or the teachers' knowledge and skill with the graphing calculator or the teachers' beliefs about the efficacy of or kinds of uses of graphing calculators in mathematics learning. In their review of the research, Wilson and Krapfl (1994) suggest that there is a need to better understand teachers' conceptions and a need 'to focus on the qualitative aspects of knowledge construction of students using graphing calculators' (p. 261). As Penglase and Arnold (1996) point out in their extensive review of the literature, many studies fail to distinguish carefully between the tool and the instructional context and assessment procedures, leading to inconsistent findings regarding the effectiveness of graphing calculators in the learning of functions. These reviewers suggest the need for studies which attempt to address graphing calculator use within particular learning environments.

In a study of teachers' roles when using graphing calculator technology in a high school pre-calculus class, Farrell (1996) found that there was an

overall shift in teachers' roles from task setter and explainer to consult-
ant, fellow investigator, and resource. These shifts occurred within each
class as the teachers moved from not using the graphing calculator and
computer technology to the use of it; the shifts were also accompanied
by a decrease in lecture and an increase in group work. Contrary to these
findings, other research has found that teachers did not change their meth-
ods or approaches when using the graphing calculator in teaching about
transformations of the parabola, except to provide more visual examples
(Simmt, 1997). The teachers in Simmt's study did not use the graphing
calculator to facilitate discussion, encourage students to conjecture or to
prove ideas. The finding that these teachers did not shift their role when
using the graphing calculator may be accounted for in part by the short
period of time they used the technology, 4 to 10 class periods (Simmt,
1997, p. 273). The teachers in Farrell's study (who had shifted their roles)
were observed using the graphing calculator near the end of a year long
course. However, the more important variable in influencing the teacher's
role with the graphing calculator may well have been the teachers' attitudes
and beliefs about mathematics and mathematics education.

This relationship was explored in a study by Tharp, Fitzsimmons and
Ayers (1997) who found that teachers who held a less rule-based view-
point about learning mathematics were more willing to adopt the use of
calculators as an integral part of instruction than those teachers who held a
rule-based view of mathematics learning. These researchers found that the
non-rule-based teachers used more inquiry learning in their classrooms and
that their students freely used the calculators as they wished. The teachers
in Simmt's (1997) study were described as having a view of mathematics
that 'strongly favors 'traditional' approaches to problems' (p. 287), sug-
gesting that they may be more rule-based and less likely to adopt a more in-
tegrated use of graphing calculator technology. The findings of Tharp et al.
suggest that there is an important relationship among the teacher's know-
ledge and beliefs, pedagogical strategies, and how students use graphing
calculators.

In this paper, we report the results of a qualitative, classroom-based
study that examined (1) the role, knowledge and beliefs of a pre-calculus
teacher, (2) how students used graphing calculators in support of their
learning of mathematics, (3) the relationship and interactions between the
teacher's role, knowledge and beliefs and the students' use of the graph-
ing calculator in learning mathematics, and (4) some limitations and con-
straints of the graphing calculator technology that emerged within the class-
room practice.

2. THEORETICAL FRAMEWORK

The theoretical framework guiding this research follows a perspective in which the psychological aspects of learning are coordinated with the social aspects though students' interactions with mathematical tasks, with each other and with their teacher within the social context of the classroom (Cobb and Yackel, 1996; Meira, 1995; Roth and McGinn, 1997). For our purposes, critical aspects of this social context include the tools and the norms for tool usage which emerge as students and teacher interact with the tool and with each other. It is through these interactions that the meaning of the graphing calculator as a tool for mathematical learning within the classroom is constructed by both teacher and students. As Hiebert et al. (1997) have observed, 'Students must construct meaning for *all* tools. . . . As you use a tool, you get to know the tool better and you use the tool more effectively to help you know about other things.' (p. 54, emphasis added). Hiebert et al. argue that meaning is constructed *for* the tool as it is used and that learners construct mathematical meaning *with* the tool. The meaning that is constructed for the tool does not precede the tool's use in constructing mathematical meaning, but rather the meaning for the tool and the meaning with the tool are intertwined through their use by participants with mathematical tasks (Meira, 1998).

As Lave and Wenger (1991) have argued, the meaning of any tool is intricately tied to the cultural practices within which it is used for some social purpose. The features of a tool are not something in and of themselves, but rather are constituted by the actions and activities of people. This view has serious consequences for interpreting the results of much of the research on graphing calculator use, which often appears to take the features and nature of the tool to be given or self-evident, as the details of how and why the tool was used are often not reported. In contrast, we seek to describe the emergence of the features of the tool in the practice of the participants as they interacted with each other and with mathematical tasks. For this study, the essential question is how do students and their teacher interpret and make use of the graphing calculator as a tool that is part of a specific cultural practice, namely their mathematics class?

In this study, we seek to describe how one teacher's knowledge and beliefs about the graphing calculator were reflected in her pedagogical strategies. We then describe how these strategies led to the co-construction, with the learners, of a particular set of ways in which the graphing calculator became a tool for mathematical learning. We closely examine how the students opted to use the graphing calculator to support their mathematical learning throughout their year-long pre-calculus course. Lastly, we discuss

our findings on how the graphing calculator, as with any tool or technology, enabled and constrained both pedagogical practices and student learning.

3. METHODOLOGY AND DATA ANALYSIS

A pre-calculus curriculum based on modeling problems in an enhanced technology environment (using graphing calculators, calculator-based measurement probes for motion, temperature and pressure, and computer software) provided a rich setting for studying the role of the teacher and the patterns and modes of graphing calculator use by the students and the teacher. This classroom-based, observational case study of two pre-calculus classes took place in a suburban school. The classes were taught by the same teacher and each met for five sessions for a total of 270 minutes per week. One class had 17 students and the other 14 students, all between 15 and 17 years of age. This selection of a single teacher in two similar classes allowed us to focus in-depth on the role and knowledge of the teacher and the interactions with and among the students. The teacher had 20 years of teaching experience and was skilled in the use of the graphing calculator. The classes were observed over three units of study on linear functions (four weeks), exponential functions (ten weeks) and trigonometric functions (seven weeks).

All of the students had either TI-82 or TI-83 graphing calculators. These devices are rich in graphing and statistical functionality, although lacking in symbolic algebra capability. In general, the students had used their calculators for well over a year before taking this course and were quite familiar with its functions. The classroom was equipped with a computer, printer and a 'graph link cable' that could be used to transfer pictures of the calculator graph to the computer for printing. The link cable also could be used to transfer data and programs between the computer and calculator, but this feature was rarely used. On the other hand, students readily transferred data and programs between calculators using the calculator-to-calculator link cable. The classroom was equipped with a view screen that allowed the calculator screen (but not the keystrokes) to be projected using a standard overhead projection unit.

Classroom instructional activities regularly alternated between modeling problems investigated by the students within a small group and whole class discussion for sharing progress, discussing solution methods and extending results. The instructional tasks were designed so that the students would create quantitative systems which describe and explain the patterns and structures in an experienced situation and which can be used to make predictions about the situation (Doerr and Tripp, 1999). The students were

asked to interpret data, to find meaningful representations of the data (typically tables, graphs and equations), and to generalize relationships beyond the particular situation at hand. The concept of the rate of change of a function and the transformations of exponential and trigonometric functions were addressed throughout the problem situations.

All class sessions were observed by two or more members of the research team. Extensive field notes, transcriptions of audio-taped group work, transcriptions of video-taped whole class discussion, and interviews and planning sessions with the teacher constituted the data corpus for this study. In an observational study such as this, there are necessarily some difficulties in observing students' use of the graphing calculator. As Williams (1993) has pointed out, it is not easy to see the students' calculator screens, much less to capture the keystrokes they are using. However, the mathematical investigations were specifically designed to be completed by small groups of students and then later shared in a whole class discussion. This format generated discussion within the small groups where students explained to each other what they did on their calculator, or one student would show another how to do a particular task, or students would compare graphical or numerical results. In this way, students' calculator use was made visible to the researchers. However, there were occasions where it simply was not clear what the students had done with their calculators. The use of the graphing calculator in the whole class setting was much easier to observe, since the teacher and the students regularly used the overhead projection view screen and explained what they were doing as they were doing it.

The data analysis was completed in two phases. In the first phase, these data were analyzed and coded for the patterns and modes of graphing calculator use by both the teacher and the students throughout the instructional units. Upon completion of this initial coding, a descriptive profile of the teacher, including her knowledge, role in the class, and beliefs was compiled. This profile was shared with the teacher for her corroboration and feedback. The initial coding of the data on the patterns and modes of student use led to the creation of five categories of graphing calculator use throughout the three instructional units. The few discrepancies in coding between the researchers were resolved by comparing the episode in question to similarly coded episodes and to other episodes involving the same students. Once we completed the initial set of categories we refined our descriptions within each of these categories by re-examining the field notes and transcripts.

In the second phase of the data analysis, we sought to refine the teacher profile by re-examining the evidence in the field notes and transcripts in

light of her feedback. Since the teacher substantially corroborated the pro-
file, this led to only minor changes in that profile. With our refined set
of categories of graphing calculator use, we sought to identify limitations
that the technology imposed on the teachers' pedagogical practices and on
the students' use of the tool to support their learning. This phase of the
analysis yielded a detailed description of the limitations and constraints of
the tool. We then completed our analysis by identifying the links between
the teacher's role, knowledge and beliefs and the students' patterns and
modes of calculator use.

4. RESULTS

We begin with a brief description of the role, knowledge and beliefs of
the teacher as they were reflected in her pedagogical strategies and in her
interactions with the students. We then present the patterns and modes of
graphing calculator use that emerged as the students interacted with the
teacher, with each other and with the mathematical tasks. Finally, we dis-
cuss our findings as to how the graphing calculator enabled and constrained
the students' learning and the teacher's pedagogical practice.

4.1. *The teacher*

The teacher was particularly skilled in using the graphing calculator, as
was demonstrated throughout the instructional units by her own use of the
calculator and by her ease in answering the students' occasional questions
on a particular calculator procedure. The teacher was familiar with the
programming features of the graphing calculator and she had written a
short program that would calculate the average slope from an ordered list
of ordered pairs of data. She felt comfortable reading programs that the
students would occasionally write. The teacher had also devised her own
method for finding a numerical approximation of derivative of any given
function. Since this was a pre-calculus course, she did not want to use
any 'black box' or built in functions of the graphing calculator to find the
derivative, but rather she wanted to make the creation of the rate of change
function explicitly visible to the students through the use of the familiar
slope formula.

   The teacher's confidence in her knowledge about the calculator's cap-
abilities and its potential uses for student learning was also reflected in
the willingness with which she encouraged the students to freely use the
calculator in their work. She actively encouraged the students to take over
the use of the calculator on the overhead projection unit during class dis-

cussions. The teacher herself showed flexibility in her use of the calculator. While using the overhead projection unit, if a student suggested an alternative viewing window or another section of a table to examine or a similar expression for comparison, the teacher was willing to take the student's suggestion.

The teacher believed that the calculator presented certain mathematical limitations. She raised the issue of the validity of the calculator results by asking questions such as 'Does the calculator always tell the truth?' and 'To what extent should we believe the calculator?' For example, in one task, the students were investigating a decay situation where they began with a small cup full of small, disk-shaped candies with an 'M' on one side. The candies were spilled onto a table and those with the 'M' showing were removed on each trial until none remained. (See Doerr (1998) for a fuller description of this mathematical task.) The students concluded that an exponential function models this decay process. One student observed that even though in their experiment they ended up with zero candies, the exponential model did not attain a zero value since 'you can divide by two infinitely without getting zero'. But another student, who was manipulating the calculator on the overhead screen, scrolled down the table for their exponential function until, for very large values of x, the function appeared to reach the value zero. This generated considerable discussion among the students about the calculator having limitations and 'not always telling the truth'. The students began to see the calculator as a tool that should be checked based on their own understandings of mathematical results.

The teacher believed that the calculator would be a helpful tool for the students to use in finding meaningful responses to problem situations. Two specific calculator-based methods (regression analysis and curve fitting by modifying parameters) were regularly used by three of the 31 students to solve problems where part of the task was to find an equation of a function to represent the data set for a given phenomena. These two methods did *not* become very popular among the students as a whole, but rather remained in use extensively by just three students. The teacher did *not* explicitly discourage these methods by telling the students not to use them, but rather she required a meaningful explanation of how the numerical results related to the problem situation. This meaningful interpretation of the result could not be given by the students who used either regression analysis or a curve fitting approach. These students did not see their findings as estimates of a mathematically determined model or of particular parameters directly related to the problem situation. Those students who used the calculator's regression functions were focused on the immediacy of obtaining some numbers (coefficients) to use in an equation and not on making sense of

TABLE I

Patterns and modes of graphing calculator use

Role of the Graphing Calculator	Description of Student Actions
Computational Tool	evaluating numerical expressions, estimating and rounding
Transformational Tool	changing the nature of the task
Data Collection and Analysis Tool	gathering data, controlling phenomena, finding patterns
Visualizing Tool	finding symbolic functions, displaying data, interpreting data, solving equations
Checking Tool	confirming conjectures, understanding multiple symbolic forms

the result's meaning through a more mathematical analysis of the problem situation. The teacher's belief that the calculator could be a helpful tool to find meaningful representations of problem situations led to a de-valuing of the regression coefficients that were easily generated by the calculator and of curve fitting by modification of parameters.

4.2. *The students*

Through our analysis of the data, we identified five categories of patterns and modes of calculator use by the students (see Table I). The focus of our analysis is on how and why these features were used in the problem contexts and instructional situations. The graphing calculator can in certain situations take on more than one role simultaneously.

   We will carefully describe each of these roles of the graphing calculator and its relationship to the teacher's role, knowledge and beliefs. We will then present the findings on the limitations and constraints of the graphing calculator in this particular teaching and learning environment.

4.3. *The calculator as a computational tool*

As a computational tool, the graphing calculator was routinely used by students to evaluate numerical expressions. Two computational issues arose: (1) correctly entered symbols and parentheses, and (2) the precision of the computational results. When incorrectly entering symbols, the students encountered a variety of error conditions. For example, when one student entered 'log(−1)', an error message appeared and the student had to acknowledge the error and explain the nature of it. This student explained the calculator's error message by referring to the domain of the logarithm

function and to the accepted mathematical fact that the logarithm of a negative number does not exist. We interpret that this call for an explanation for calculator results (whether errors or not) had become part of the mathematical norms established by the teacher, whereby the accepted truth or falsehood of a statement had to be supported by mathematical reasoning or justification, not by an appeal to any authority ascribed to the calculator.

A second computational issue emerged as the students attended to the need for rounding (or approximating) the results given by the calculator. The need for an approximation of the numerical results given by the calculator was closely linked to the interpretation the students gave to the problem situation. Their interpretation of the constraints and restrictions imposed by the context of the problem determined whether and how they saw the range of the possible numerical values of the result. This happened, for example, in recursive discrete processes of exponential growth and decay. One group of students discussed whether to round the result at each stage or at the end of the process in a population growth problem. They argued that the fractions at each stage could add up to an integer at the end and that therefore this could affect the final size of the population. For price discount problems, the students relied on their common sense knowledge of store prices and rounded the discounted prices to the nearest dollar, but only at specified time intervals. They knew that a discount would not be made daily, but more likely at the end of each week. The mathematical meanings of the numerical results were interpreted by the students so as to make descriptive sense of the experienced phenomena.

While the students attended to the issue of rounding in the problem situations just described, there were other situations in which the students simply worked with whatever number of decimal places the calculator happened to provide. For example, the students readily worked with numbers having many decimal places that represented measurements for distances generated by a motion detector, water temperatures measured by a temperature probe, pressure sensed by a pressure belt and meter, and heights calculated using the sine function. The students would record and calculate with these numbers, even though they appeared to recognize that 'real-life' measurements could not be taken to 6 or 8 decimal places of accuracy. They showed no tendency to round the numbers and provide more 'realistic' answers. On the contrary, the large number of decimal places appeared to be taken as the more realistic answer.

4.4. *The calculator as a transformational tool*

One of the most significant uses of the calculator in this setting was as a transformational tool, whereby tedious computational tasks were trans-

formed into interpretative tasks. As we noted earlier in our profile of the teacher, this teacher regularly asked students about the meaning of the coefficients and constants in any equations they used. By her focusing the students' attention on the interpretation of the result, rather than on the actual computation, the students attended to making sense of the result and validating it in the context of the problem situation. This transform-ation of tasks from computational to interpretative was especially evident throughout our analysis of the students' investigations of several rate of change problems.

As a computational task, determining the rate of change for a given function for small variations of the independent variable on the entire do-main would be tedious to perform without a calculator. Using a paper and pencil approach for this task would restrict the study of the rate of change to a local property instead of the global view of the rate of change, which in turn leads to an understanding of the rate of change as a function itself. Many students used their graphing calculator to construct 'rate functions' of the form $y_2(x) = [y_1(x+\Delta x) - y_1(x)]/(\Delta x)$, where $y_1(x)$ is the original function and Δx is an interval for the independent variable. The students usually set this value to .1 or .01. This syntax for the rate function had been devised by the teacher and introduced to the students through the use of the familiar formula for slope. This form was initially used with standard function notation and then became instantiated in the calculator notation given above.

The students and the teacher continued to use both paper-and-pencil and the calculator for estimating the rate of change of a function. However, it was the calculator-based form of the numerical estimate of the rate of change of a function that transformed tasks from a computational focus to an interpretative one. For example, the students were given the task of finding a function to describe the vertical position of a point on a rotating Ferris Wheel, relative to the hub of the Ferris Wheel. The students were also asked to find a function to describe the corresponding rate of change in the vertical position with respect to time. Nearly all groups of students used the 'rate function' to construct the rate of change function, y_2, after they had found a suitable expression for the vertical position function, y_1. This computation of the rate function generated the table of values for the rate-function and its corresponding graph. From this, they were able to validate in terms of their experience with the physical models of the Ferris Wheel the periodicity of both the vertical position function and the rate of change of that function. This linked the interpretation of the resulting equation with the experienced phenomena and shifted the focus of the task from the

details of each pairwise computation for rate to a global interpretation of the function over its domain and in relationship to the problem's context.

4.5. *The calculator as a data collection and analysis tool*

During activities with Calculator Based Lab (CBL) devices, motion detectors, pressure belts, temperature probes, and special programs, the calculator was used as a tool for data collection and analysis. The collected data was stored, compared, and re-collected until the students decided that they had acquired a 'satisfactory' data set. Through this kind of use of the tool, students needed to engage in understanding the context of the activity and in deciding, through a process of conjecturing, refining, and negotiating, what constituted a 'satisfactory' set of data. In several of these activities, we found that students repeated the same experiment many times, until the set of data they had collected graphically matched an expectation they had about the behavior of the phenomena. There were two distinct modes of repetition that emerged. On the one hand, in working with temperature probes and the motion of a bouncing ball, the students acted with relatively little direct control over the gathering of the data. In this case, the students would simply repeat an experiment hoping to acquire a data set that more nearly matched their expectations. On the other hand, there were several activities that evoked a much greater sense of direct control over the phenomena.

In one such activity, the students were asked to use a pressure belt to gather data that represented the pattern of 'normal' breathing and to find a function that could be used to describe that pattern. Since many of the students expected to obtain a sine-type plot for their data, they purposefully adjusted their breathing in terms of both the amount of air inhaled and the rate at which it was inhaled. They attempted to control their breathing and hence the data collection device and the resulting data in such a way that the data were closer to expected shape and numerical characteristics (i.e. without jagged peaks and almost constant amplitude). In this way, the graphing calculator and the attached pressure belt became a tool that supported the students in controlling the data collection through cycles of interpretation of the graphical results and purposeful changes to the physical phenomena of breathing.

4.6. *The calculator as a visualizing tool*

As a visualizing tool, we found that the students used the graphing calculator in four ways: (1) to develop visual parameter matching strategies to find equations that fit data sets, (2) to find appropriate views of the graph and determine the nature of the underlying structure of the function, (3) to

link the visual representation to the physical phenomena, and (4) to solve equations. Whenever the students had a set of data, they were quick to plot the data; this inclination to visualize data could be seen throughout all three units, but most especially in the exponential and trigonometric units. A small number of students frequently used a visual parameter matching strategy when they were faced with the task of finding an equation that describes a given data set. The approach taken is illustrated by one student in his attempts to find an equation for the harmonic motion of a spring, gathered from a motion detector. This student, basing his strategy on the recognition of the class of trigonometric functions and his knowledge about the symbolic representations of transformations, started with an equation of the type $y = A \sin(B(x + C)) + D$. This student systematically varied the parameters A, B, C, and D in his equation in order to obtain a visual fit between the plot of the data set and the graph of the function.

The students became very skilled at determining appropriate viewing windows for data sets and for symbolically defined functions. Sometimes they used trial and error approaches to find better viewing windows, but most of the time they set the window by using specific entries from the table values of the relationship to be graphed. They developed the realization that a quick judgment based on the shape of a graph was not enough for deciding the nature of the function. The students had seen situations in which an exponential shape can be visually mistaken for a quadratic shape and when a close zoom in on a function made it look linear. As a consequence of this and of the teacher's belief that the calculator itself cannot provide the authority for a mathematical argument, the students found reasons other than appearance to justify the underlying structure of the function.

The graphing calculator was used as a visualizing tool when the students examined the discrete data values describing a phenomenon and had to relate the data back to the continuous physical process and the function which potentially describes that process. For example, in the Ferris Wheel investigation, a group of students constructed a table for the rate of change using their 'rate function' and observed that what appeared to be the highest value of the function repeated consecutively. Knowing, from their experience of problem situation, that the motion of the wheel was continuous, they decided that 'there was something in between'. The students then shifted from the table to the algebraic expression for the continuous graph and traced it, thereby confirming their conjecture and finding the single highest value. The graphing calculator provided a visual representation of the continuous function that described the rate of change in the vertical motion of the Ferris Wheel. This became the link between the

discrete data table, which was missing a critical data point (the maximum value), and the physical phenomena, which, from their experience with the wheel, the students conjectured must reach some unique maximum value.

The graphing calculator's use as a visualizing tool was also reflected in how the students solved equations or inequalities. In the class discussions, the teacher had discussed all of the available methods for solving an equation and encouraged the students to choose among these methods. The methods included paper and pencil solutions, use of the calculator's 'Solve' command, and graphical solutions. We found that many of the students used a graphical approach, which involved less computation and appeared to support a more meaningful interpretation of solutions. For example, in one problem, the students were asked to determine when a bank account attained a given amount of money. Many students approached this by examining the intersection of the graph of the function describing the amount of money at any point in time with the graph of the horizontal line representing the given amount. This visual approach seemed to support their description of the amount of money in the bank reaching a given value.

4.7. *The calculator as a checking tool*

In this study, we considered the use of the calculator as a checking tool when it was used to check conjectures made by students as they engaged with the problem investigations. This was followed by rejecting and re-conjecturing, trying to prove the conjecture, or simply accepting the conjecture. In many tasks, the students posed a conjecture about a possible function as fitting a data set and then they used their graphing calculator to check how well it fit. The students' strategies depended in part on how they had found an equation in the first place. For example, in the case of the student using visual parameter matching for find an equation for the data from harmonic motion (described above), the visual mis-match between his conjectured equation and the data was crucial for his decision to reject his equation and try a new one. In this way, the graphing calculator was an essential check of the match between his equation and the data. In other cases, we found that the calculator was used as a checking tool only in the most trivial sense. For example, when one student found an equation using the regression features of the calculator, he checked the match between the regression equation and the data. Since the students generally chose an appropriate regression model, the graphical mismatches were generally minor. Those few students who preferred regression equations were inclined to 'go with what the calculator says' and rarely questioned the fit of the equation to the data.

When the students had determined the function through a meaning-ful mathematical process, they would check to see how well their graph matched the data. Occasionally, a graphical mismatch revealed a mistake in entering the form of the equation or in a computation. For example, in finding an equation for the harmonic motion described earlier, one student determined the amplitude, both the horizontal and vertical shift, and the frequency by tracing the coordinates of consecutive maximum points in the data set. This approach enabled her to find a meaningful equation based on her knowledge of trigonometric functions and their transformations. In this case, the check between her conjectured equation and the data set was confirmed as she observed that the graph and the data were visually aligned along critical features such as local maxima and minima.

The calculator's role as a checking tool was evident in the context of activities on transformations on functions, which were designed both in go-ing from the equation to the graphical representation and from the graph to finding an equation (non-unique) to match the given graph. The calculator was initially used as a visualizing or graphing tool, but as students grasped the ideas of transformations, the calculator was used almost exclusively as a checking tool. The graphing calculator helped the students understand the idea of the non-uniqueness of algebraic representations for the same graph in the case of exponential and trigonometric functions. For example, the students were asked to compare the graph of $y = 8 * 2^x$ with the graph of $y = 2^{x+3}$ and found that the graphs appeared identical. Using the overhead display unit, the teacher switched to the table of values, revealing that the numerical values were also identical. This then led to an algebraic argument that supported the claim that these two functions were identical. In this way, the graphing calculator was used to confirm conjectures, but at the same time, the teacher led the mathematical discussion to the need for justification by algebraic reasoning.

The role of the graphing calculator as a checking tool by both the students and the teacher was especially interesting in the case of the trigo-nometric functions. The periodicity of the trigonometric functions results in a particularly salient characteristic, namely the 'look alike' feature of the graphs of these functions. One can argue that the 'look alike' appearance on the screen of the calculator is common to almost any class of functions, certainly to the exponential class that was studied by these students. How-ever, for periodic functions, it is the case that simple horizontal shifts of the viewing window can lead to no visible change in the appearance of the graph and this characteristic is not true for other functions. Difficulties in interpreting a periodic graph were increased by the limitations of the graphing calculator's screen, where the units on the coordinate axes are

not labeled. The investigation of trigonometric functions with the calculator became limited to the use of the tables and occasionally the 'Trace' command. This pattern of use as a checking tool for the trigonometric function was limited by a mismatch between the decimal representations of the calculator and many of the tasks where the values of the independent variable were often given as rational multiples of π. The calculator's table and the 'Trace' command showed these values in decimal form. Hence, there was no easy direct comparison of the closeness of numerical values without an extra tedious conversion step of the value of the rational multiple of π to a decimal value.

4.8. *Constraints and limitations with the graphing calculator*

The graphing calculator emerged as a constraint and limitation in two ways: (1) students' attempted uses of the device as a 'black box' without attending to meaningful interpretations of the problem situation; and (2) the personal (or private) use of the tool. The use of the calculator as a 'black box' represents the class of situations in which the students did not have a meaningful strategy for the use of the calculator. In one particular task, the students were asked to design simulation of the spread of a rumor. The students were given the calculator syntax for generating single random numbers and lists of random numbers. Some groups of students decided that they had to use the random number generator and tried to do so before they actually read the content of the activity and understood what the random number syntax might be useful for. These students did not understand that the calculator's random number generator was simply an alternative tool, not the only tool, and certainly not 'the solution' to the problem. At least one group of students became considerably sidetracked on the quantitative details of how the random number worked. They spent considerable time discussing if they needed to correct for whether or not a zero could be generated by the random number generator. This discussion occurred well before they had found a use for random numbers in the design of their simulation.

Another limitation of the graphing calculator was in its use as a private device. Unlike the case of group work with a computer, where the screen is a visible and hence potentially shared artifact, the calculator's screen was not visible in a shared way and was not necessarily common among a group of students. The tendency of the students to use their calculators as private devices regularly led to the breakdown of group interactions. We found a pattern that began when two or more students in a group tested or checked a possible conjecture or computation on their own calculators and then continued to use the tool to explore possible solutions, interpret-

ations or refinements of their own thinking. The results of the graph, table, or computation were used to further their own individual thinking, rather than shared back with the group. This closed the communication network among the students and made the re-opening of discussion quite difficult. As time progressed, the students had different interpretations, representations and problems rather than a common or shared view of the task at hand. Once this closing of communication had happened within a group, the students tended to continue to pursue their investigations as individuals, with little talking and no sharing of ideas, representations, or results. If the students had questions, they would ask them of the teacher, rather than each other.

This use of the calculator as a private, personal tool was in sharp contrast to its use during the class discussion, as a shared device (via the overhead screen). This shared, public display was supportive of the communication among students and was used to facilitate the comparison and unification of ideas. Since the teacher let the students control the calculator in most instances, this shared device helped encourage student initiative and often resulted in the students' leading of the discussion. The decision to use the calculator as a shared device in the classroom came from the development of the discussion and was often student-requested.

5. DISCUSSION

The role, knowledge and beliefs of the teacher and the actions of the students as they interacted with each other and with the graphing calculator created meaning for the tool and meaning with the tool. The teacher's confidence in her own knowledge and skills and her own flexible use of the calculator led to a classroom environment where students were free to use their calculators as they wanted and were actively encouraged to use them to calculate, explore, confirm, or check mathematical ideas. The teacher's knowledge of the limitations of the calculator led her to encourage the students to question their calculator-based results. In this way, the calculator became a tool that itself needed to be checked on the basis of mathematical reasoning. Contrary to the concerns raised by Williams (1993) and Wilson and Krapfl (1994), the calculator did not become a source of mathematical authority in this classroom. We see this as a consequence of the teacher's knowledge of the limitations of the calculator and her belief that conjectures are proven on the basis of mathematical reasoning or argument.

The teacher's belief that the graphing calculator would be a helpful tool for the students to use in finding meaningful responses to mathematical

tasks is evidenced by the kinds of questions she asked the students. She regularly asked the students about the meaning of the coefficients that they found and how those coefficients related back to the experienced phenomena. We found that in finding equations to fit graphs or numerical data sets, the most common strategy used by the students was a meaningful mathematical approach. The visual-parameter matching strategy, while based on the mathematical recognition of the general form of the function, was used extensively, but by only a few students. These findings confirm the results found by Ruthven (1990). Furthermore, for the trigonometric functions, we found that the use of the graphing calculator as a checking tool to confirm their conjectures was not used by most students, but rather they relied solely on their algebraic and geometric knowledge of the transformations.

The teacher's belief in meaning making led to the de-valuing of regression equations as solutions and limited the 'black box' use of the graphing calculator. This was particularly visible in the teacher's strategic choice to use a rate function that was built upon the students understanding of the familiar slope formula rather than a use of built-in derivative functions. The rate function served to illuminate a powerful role for the graphing calculator as transforming a local, computational task (the computation of average rate of change) to a global, interpretative task (the relationship between the numerically determined rate of change function and the given function over the entire domain).

The role of the graphing calculator as a device that can both monitor and control data collection appears particularly powerful and has been little explored in the research literature to date. In this study, we found that students engaged in two distinct actions: one where they repeated the data collection to match their expectations and the other where they controlled their actions to match their expectations. As a visualizing tool, the students used the graphing calculator to find equations that matched data sets, to find appropriate views of the graph, to link the visual representation of the graph to physical phenomena in explanatory ways, and to solve equations.

The most significant limitation of the calculator was as its use as a private device. While we did observe, as did Farrell (1996), that students frequently used their calculators while the teacher or other students were talking in lecture or whole class discussion, we also observed that this personal use of the technology served to breakdown group communications. Once the students began to work individually on a task, it was very difficult for them to resume functioning as a group, since their thinking about the problem had often progressed in different directions. In contrast, the use of the overhead projection unit appeared powerful in generating shared representations, alternative interpretations, and contrasting conjectures.

Conclusions

The graphing calculator has been the focus of numerous research studies that have sought to answer the question of whether or not the use of the graphing calculator is effective in achieving learning goals for students. However, such studies do little to illuminate the patterns and modes of graphing calculator use by students or the roles, knowledge and beliefs of teachers or the influence of the mathematical tasks on the students' activities. In this study, we have analyzed and described how students and their teacher interpret and make use of the graphing calculator as a tool that is part of their mathematical practice.

We found that five patterns and modes of graphing calculator tool use emerged in this practice: computational tool, transformational tool, data collection and analysis tool, visualizing tool, and checking tool. This suggests that the graphing calculator is a rich, multi-dimensioned tool and that the continued study of its use in classroom practice will need to carefully delineate the patterns and modes of use that occur in any given context as conclusions are drawn related to student learning. The results of this study suggest that the role, knowledge and beliefs of the teacher influenced the emergence of such rich usage of the graphing calculator. The teacher's role in encouraging interpretation and explanation led to a valuing of meaningful mathematical constructions for equations, to the transformation of rate from a computational task to an interpretive task, to the valuing of algebraic arguments to support graphically and numerically generated conjectures, and to a de-valuing of regression equations or appeals to the calculator as an authority in a mathematical argument. We found that the calculator as a private device inhibited mathematical communication in a small group setting, while the shared screen of the graphing calculator appeared to be a powerful tool for supporting the comparison and unification of mathematical ideas.

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HELEN M. DOERR and ROXANA ZANGOR
Department of Mathematics
Syracuse University
Syracuse, New York
U.S.A.
E-mail: hmdoerr@syr.edu

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