Fractions Section 1: Iterating and Partitioning

Goal

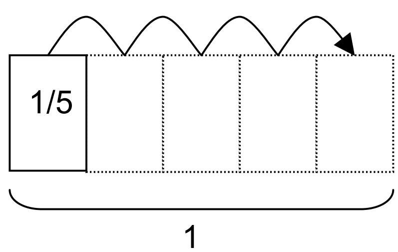
To help students develop iterating and partitioning images for fractions that they can use to (1) reason about part-whole relationships, and (2) describe, both orally and in writing, what a fraction means.

Big Ideas

Fractions are often taught using whole number ideas and language, which leads children to think about fractions as two whole numbers located vertically in space, not as a single, meaningful entity. In this section, I present two different images for fractions that are uniquely different from whole number reasoning, namely iterating and partitioning.

Iterating

One image for thinking about fractions is iteration. To conceive of a fraction from an iteration perspective, first start with a unit fraction, such as 1/5. How can you tell if something is 1/5? An amount is 1/5 if five copies of it equals 1. The image here is of taking 1/5 and iterating it 4 more times to make a whole.



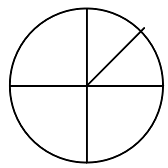
The power in this image is that it gives a way to check to see if something is 1/5. Students can use this image when reasoning about fractions to make sure that they are attributing the correct size to fractional pieces. It also emphasizes that fractions are always a fraction of something. 1/5 is 1/5 of what? Of 1. How do you know? Because five copies of 1/5 is 1.

Once we understand what unit fractions are from an iterative perspective, then we can talk about the meaning of non-unit fractions, such as 4/5. Four-fifths means four one-fifths, where one-fifth is understood to be the amount such that five copies of that amount, when combined, equal a whole.

Iterating vs. "Out of" Thinking

You may have difficulty thinking about fractions in terms of iterations because you may have thought about fractions mostly from an "out of" perspective. For example, you may think of 1/5 as 1 out of 5 things or parts. The problem with this image of fractions is that it is really whole number thinking. You are thinking of the fraction as two whole numbers, 1 and 5. It doesn't require you to use any fraction ideas. The "out of" image of fractions leads to several breakdowns, including some of the following:

- Lack of uniformity in size. All of the pieces below are 1/5, because they are each 1 out of 5 pieces.



- It obscures that fractions are always fractions of something. 1/5 thought of as 1 out of 5 is one-fifth of what? 1? 5?

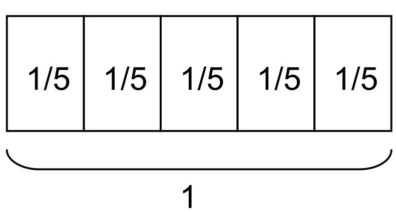
- Complex fractions don't make sense. What does 3/2 mean? 3 out of 2 things? How can you have 3 out of 2 things?

- Lack of relative sizes. 2/8 is bigger than 1/3, because 2 candies out of 8 candies is more than 1 candy out of 3 candies.

- Obscures the actions of iterating and partitioning (partitioning discussed below). When children say something like "3 out of 5 equal parts" to explain 3/5, they are not thinking of performing an action, like cutting up a whole. They are missing the actions that are used to create the fraction, and that are an essential part of understanding what the fraction means.

Partitioning

A second image for thinking about fractions is partitioning. To conceive of a fraction from a partitioning perspective, first start with a unit fraction, such as 1/5. How can you tell if something is 1/5? It is 1/5 if it is the size of a piece you would get by taking a whole and splitting it into 5 equal parts.



The power in this image is that it gives a way to generate 1/5. We get 1/5 by taking a whole and splitting it into 5 equal parts. Students can use this image when reasoning about fractions to create a fractional piece of the correct size. Like the iterating image, this image also emphasizes that fractions are always a fraction of something. 1/5 is 1/5 of what? Of 1. How do you know? Because it is created from 1 by partitioning 1 into five equal pieces.

Once we understand what unit fractions are from an partitioning perspective, then we can talk about the meaning of non-unit fractions, such as 4/5. Four-fifths means four one-fifths, where one-fifth is understood to be the amount that one gets from partitioning 1 into 5 equal pieces.

The image of partitioning is a welcome addition in reasoning about fractions. For example, if a student only has the iteration image to work with and is asked to determine how much 1/5 of a particular amount would be, they would have to use the guess and check method to find the amount. In other words, they would have to guess an amount, iterate it to see if it worked, and if it didn't, continue to modify the guess and iterate to check until a close enough approximation is achieved. With partitioning, the student has a direct method for creating 1/5: divide the amount into 5 equal parts.

The images of partitioning and iterating are very compatible. In fact, to function well with fractions, both images are necessary. Partitioning helps us be able to create fractions of different sizes, and iteration can be used to check whether we have achieved the right fraction by iterating and comparing with the whole. For example, we create 1/5 by partitioning 1 into 5 equal parts, and then check that it is 1/5 by iterating one of the parts 5 times to get 1. In this unit on fractions, one image should not be stressed over the other. You should become flexible in using both perspectives.

Partitioning vs. "Out Of"

You may have difficulty distinguishing between fractions from a partitioning perspective and the more common (but problematic) "out of" perspective. Because of this difficulty, I purposely started our discussion of what fractions mean with the iteration image, because this is so different from the "out of" perspective so as not to be easily confused. Partitioning and iterating images are fundamentally different ways of conceiving of fractions from the "out of" perspective.

The key difference between the partitioning and "out of" perspectives is that partitioning always keeps the partitioning **process** in the foreground. For example, when making fifths, we start with a whole and partition it into 5 equal parts to yield fifths. From the "out of" perspective, we don't really care where the five parts came from, what they were five parts of, and maybe not even that they are five equal parts. We are merely interested in taking a whole number amount from a whole number total. In fact, we never have to conceive of the five things as a whole whatsoever. They can remain five things. Because the partitioning image never loses sight of the partitioning process, it also never loses sight of the part-whole relationship, and thus represents fraction reasoning, not whole number reasoning.