8–3A Graphing Technology

Polynomial Functions

You can use a graphing calculator to graph polynomial functions and approximate the real zeros of the function. When using a calculator to approximate zeros, it is important to view a complete graph of the function before zooming in on a certain point. Otherwise, zeros may be overlooked because they were not in the viewing window. Remember that a complete graph of a function shows all the characteristics of the graph such as all x- and y-intercepts, relative maximum and minimum points, and the end behavior of the graph.

Example 1

Use a graphing calculator to obtain a complete graph of 

\[ f(x) = 2x^3 + 6x^2 - 14x + 12 \]

Then approximate each real zero to the nearest hundredth.

Let’s try graphing in the standard viewing window:

Enter:

\[ \text{Y} = 2 \times x^3 + 6 \times x^2 - 14 \times x + 12 \]

\[ 6 \times x^2 + 14 \]

We see that this viewing window does not contain a complete graph. Change the viewing window to \([-10, 10]\) by \([-10, 60]\) with a scale factor of 1 for the x-axis and 5 for the y-axis.

This window can accommodate the complete graph.

According to the graph, there is one x-intercept (real zero) for this function. There are three zeros for any third-degree polynomial, so two of the zeros for this function must be imaginary. Use ZOOM, TRACE, or ROOT to approximate the real zero.

The only real zero is approximately 

\[-4.74\].

When using a graphing calculator to approximate real zeros, it is helpful to know that a function with degree \(n\) has at most \(n\) real zeros. Thus, a function with degree 5 has at most five real zeros. If you can see five x-intercepts in the viewing window, you know you have found all of the zeros and that they are all real. However, if there are fewer than five x-intercepts, there are duplicate real zeros or the zeros are not all real. Complex or imaginary zeros occur in conjugate pairs, so a fifth-degree function may have one, three, or five real zeros.
Example

Use a graphing calculator to obtain a complete graph of \( f(x) = 3x^5 - 5x^4 - 2x^3 + x^2 - 6x + 8 \). Then approximate each real zero to the nearest hundredth.

First, try graphing in the standard viewing window.

Enter:

\[
3 \begin{array}{c}
X,T,\theta,n
\end{array} \quad \begin{array}{c}
\triangle
\end{array} \quad 5 \quad \begin{array}{c}
\triangle
\end{array} \quad 5 \begin{array}{c}
X,T,\theta,n
\end{array}
\]

\[
\begin{array}{c}
\triangle
\end{array} \quad \begin{array}{c}
4
\end{array} \quad \begin{array}{c}
\triangle
\end{array} \quad 2 \begin{array}{c}
X,T,\theta,n
\end{array} \quad \begin{array}{c}
\triangle
\end{array} \quad 3 \begin{array}{c}
+
\end{array}
\]

\[
\begin{array}{c}
X,T,\theta,n
\end{array} \quad \begin{array}{c}
x^2
\end{array} \quad \begin{array}{c}
-
\end{array} \quad 6 \begin{array}{c}
X,T,\theta,n
\end{array} \quad \begin{array}{c}
+
\end{array}
\]

\[
8 \begin{array}{c}
\text{ZOOM}
\end{array} \quad 6
\]

The standard viewing window does not accommodate the complete graph. The view shown at the right uses the window \([-5, 5]\) by \([-10, 15]\).

According to the graph, there are three real zeros for this function. Use ZOOM, TRACE, or ROOT to approximate the real zeros. They are approximately \(-1.24, 0.93, \text{and } 2\).

Exercise

Use a graphing calculator to obtain a complete graph of each polynomial function. Describe your viewing window and state the number of real zeros.

1. \( f(x) = 2x^3 - 3x^2 - 12x + 17 \) \([-10, 10]\) by \([-5, 25]\), 3
2. \( f(x) = 3x^4 - 8x^3 - 35x^2 + 72x + 47 \) \([-10, 10]\) by \([-125, 100]\), 4
3. \( f(x) = 0.1x^4 + x^3 - x^2 + 3x + 18 \) \([-15, 10]\) by \([-175, 75]\), 2
4. \( g(x) = x^5 - 4x^4 + 2x^3 - 7x + 15 \) \([-10, 10]\) by \([-40, 40]\), 3

Graph each function so that a complete graph is shown. Then approximate each of the real zeros to the nearest hundredth.

5. \( f(x) = x^4 - 3x^2 - 6x - 2 \) \(-0.41, 2.41\)
6. \( h(x) = 2x^5 + 3x - 2 \) \(0.61\)
7. \( c(x) = 3x^{13} + 4x^3 + 2 \) \(-0.78\)
8. \( m(x) = 2x^8 + 4x^2 + 1 \) no real zeros
9. \( p(x) = 8x^5 - 20x^3 + 73x^2 + 28x - 4 \) \(-2.38, -0.45, 0.11\)
10. \( f(x) = x^5 + x^4 - 8x^3 + 10x^2 + 7x - 4 \) \(2.83\)
Graphing Polynomial Functions and Approximating Zeros

**What You'll Learn**
- To approximate the real zeros of polynomial functions,
- To find maxima and minima of polynomial functions, and
- To graph polynomial functions.

**Why It's Important**
You can graph polynomial functions to solve problems involving geometry and physical fitness.

**Integration**

**Geometry**

In her geometry class, Hillary Borchers has a project in which she must create cracker shapes that form a tessellation in the same manner as some Keebler crackers. For her art class, she must design a special packaging box for the crackers. In order to best display the tessellation, the bottom of the box will be a square, and it will be open at the top so that the crackers can be seen through cellophane.

Hillary is to use a 108 square-inch sheet of a special metallic paper to cover the sides and bottom of the box. What will be the dimensions of the box if it is to hold a maximum volume of crackers? You can use a graph to solve this problem.

First, write polynomial equations to describe the surface area and volume of the box. Let \( x \) represent the length of the side of the square on the bottom, and let \( h \) represent the height of the box.

\[
\text{Surface Area} = \text{area of the base} + \text{area of the four sides} \\
SA = x^2 + 4xh \\
108 = x^2 + 4xh \quad \text{Replace } SA \text{ with } 108.
\]

\[
\text{Volume} = \frac{\text{area of the base}}{\text{height}} \\
V = \frac{x^2}{h}
\]

In order to find the volume by graphing, we need to express the volume in terms of one variable. First, solve the surface area formula for \( h \) in terms of \( x \).

\[
x^2 + 4xh = 108 \\
4xh = 108 - x^2 \\
h = \frac{108 - x^2}{4x}
\]

Then, substitute the value of \( h \) in the formula for volume.

\[
V = x^2h \\
= x^2 \left( \frac{108 - x^2}{4x} \right) \quad \text{Replace } h \text{ with } \frac{108 - x^2}{4x}. \\
= \frac{108x^2 - x^4}{4x} \quad \text{Simplify.} \\
= \frac{27x - \frac{x^4}{4}}{1}
\]

Let \( V(x) = 27x - \frac{x^3}{4} \).
Make a table of values.

<table>
<thead>
<tr>
<th>x</th>
<th>V(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-11</td>
<td>35.75</td>
</tr>
<tr>
<td>-10</td>
<td>-20</td>
</tr>
<tr>
<td>-9</td>
<td>-60.75</td>
</tr>
<tr>
<td>-8</td>
<td>-88</td>
</tr>
<tr>
<td>-7</td>
<td>-103.25</td>
</tr>
<tr>
<td>-6</td>
<td>-108</td>
</tr>
<tr>
<td>-5</td>
<td>-103.75</td>
</tr>
<tr>
<td>-4</td>
<td>-92</td>
</tr>
</tbody>
</table>

Sketch the graph of the function \( V(x) \) by connecting those points with a smooth curve. The graph will cross the \( x \)-axis somewhere between the pairs of \( x \) values where the corresponding \( V(x) \) values change sign. Since the \( x \)-intercepts are zeros of the function, there is a zero between each pair of these \( x \) values. This strategy is called the location principle.

The Location Principle

Suppose \( y = f(x) \) represents a polynomial function and \( a \) and \( b \) are two numbers such that \( f(a) < 0 \) and \( f(b) > 0 \). Then the function has at least one real zero between \( a \) and \( b \).

The plural of maximum and minimum are maxima and minima.

The graph above shows the shape of the graph of a general third-degree polynomial function. Point \( A \) on the graph is a relative maximum of the cubic function, since no other nearby points have a greater \( y \)-coordinate. Likewise, point \( B \) is a relative minimum, since no other nearby points have a lesser \( y \)-coordinate. You can also see from the tables of values where there is a relative maximum and a relative minimum. You can use this information to help graph functions that have imaginary zeros.

You can use the coordinates of the relative maximum to determine the point at which the box has the maximum volume. In the tables of values, the point at \((6, 108)\) appears to have the greatest \( y \)-coordinate. To check whether it is truly the relative maximum, compute the \( y \) values for an \( x \) value on either side of this point.

Find \( V(5.9) \) and \( V(6.1) \).

\[
\begin{align*}
V(x) &= 27x - \frac{x^3}{4} \\
V(5.9) &= 27(5.9) - \frac{5.9^3}{4} \\
&= 107.96 \\
V(6.1) &= 27(6.1) - \frac{6.1^3}{4} \\
&= 107.95
\end{align*}
\]

Since both \( y \) values are less than the \( y \) value for the maximum, the point at \((6, 108)\) is a relative maximum. So, in order for the box to have a maximum volume, the side of the box has to be 6 inches long. The box would have a maximum volume of 108 cubic inches.
To determine the height of the box, substitute 6 for \( x \) in the surface area equation.

\[
h = \frac{108 - x^2}{4x}
\]

\[
= \frac{108 - 6^2}{4(6)} \quad \text{Replace } x \text{ with } 6.
\]

\[
= 3
\]

The box must be 3 inches tall to have maximum volume. Thus, the dimensions of Hillary's cracker box are 6 inches by 6 inches by 3 inches.

**Example 1**

Under certain conditions, the velocity of an object as a function of time is described by the function \( V(t) = 9t^3 - 93t^2 + 238t - 120 \). Approximate the zeros of \( V(t) \) to the nearest tenth and draw the graph.

Evaluate the function for several successive values of \( t \) to locate the zeros. Then plot the points and connect them to form a smooth graph.

<table>
<thead>
<tr>
<th>( t )</th>
<th>( V(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-120</td>
</tr>
<tr>
<td>1</td>
<td>-34</td>
</tr>
<tr>
<td>2</td>
<td>56</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>-80</td>
</tr>
<tr>
<td>5</td>
<td>130</td>
</tr>
<tr>
<td>6</td>
<td>-96</td>
</tr>
<tr>
<td>7</td>
<td>76</td>
</tr>
</tbody>
</table>

One zero lies between 0 and 1. Another zero is 3. A third zero lies between 6 and 7.

To approximate the zeros to the nearest tenth, you have to repeat the process of evaluating \( V(t) = 9t^3 - 93t^2 + 238t - 120 \) for successive values of \( t \) expressed in tenths, as we did in the application at the beginning of the lesson. Using a scientific calculator will help find these values more easily.

To evaluate \( V(0.5) \), do the following.

**Enter:**

\[
9 \times 0.5 + 3 \times 93 + 238 \times 0.5 - 120 \times 0.5
\]

\[
= -23.125
\]

Following this procedure for the rest of the values in the chart, you will find that the zeros approximated to the nearest tenth are 0.7 and 6.7.
Example 2

Graph \( f(x) = x^3 - 5x^2 + 3x + 12 \).

In order to graph the function, you need to find several points and then connect them to make a smooth curve. Since \( f(x) \) is a third-degree polynomial function, it will have 3 or 1 real zeros. Also, its left-most points will have negative values for \( y \), and its right-most points will have positive values for \( y \).

Make a table and evaluate several successive values of \( x \) to locate the zeros and to find the relative maximum and relative minimum.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-22</td>
</tr>
<tr>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>0.5</td>
<td>12.375</td>
</tr>
<tr>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>2.9</td>
<td>3.039</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

--- zero between \( x = -2 \) and \( x = -1 \)

--- indicates a relative maximum

--- indicates a relative minimum

The function has one relative maximum and one relative minimum. The values of \( f(0.5) \) and \( f(2.9) \) were calculated to approximate the maximum and minimum more closely. There is a zero between \( -2 \) and \( -1 \). Use a graphing calculator to check the graph.

A graphing calculator can be helpful in finding the relative maximum and relative minimum of a function.

**Exploration**

To find the relative maximum and relative minimum of \( f(x) = x^3 - 6x^2 + 6x + 5 \), press \( \text{Y=} \) and enter the equation. Then press \( \text{ZOOM 6} \). The graph appears to have a relative minimum between 3 and 4 and a relative maximum between 0 and 1. To find the actual relative minimum, follow these steps.

Enter: \( \text{[MATH] 6 2nd [Y-VARS] 1 ENTER [\( \text{\(X,T\), \( \theta \), n}\)] ENTER 3 4 ENTER} \)

Thus, there is a relative minimum at \( x \approx 3.41 \).

**Your Turn**

a. Find the \( y \)-coordinate of the relative minimum to the nearest hundredth. \(-4.66\)

b. Find the coordinates of the relative maximum of the function to the nearest hundredth. (Hint: Use the \( f\text{Max} \) feature by pressing \( \text{[MATH] 7} \))

c. Graph the function \( f(x) = x^3 + x^2 - 7x - 3 \), and find the relative maximum and relative minimum to the nearest hundredth.
Communicating Mathematics

Study the lesson. Then complete the following.

1. State the greatest number of relative minima that are possible for each condition.
   a. a third-degree polynomial with a positive leading coefficient 1
   b. a third-degree polynomial with a negative leading coefficient 1
   c. a fourth-degree polynomial with a positive leading coefficient 2
   d. a fourth-degree polynomial with a negative leading coefficient 1

2. Refer to the application at the beginning of the lesson. Why did we not choose one of the negative zeros for the volume of the box?

3. Sketch a graph of each polynomial.
   a. even-degree polynomial function with one relative maximum and two relative minima
   b. odd-degree polynomial function with one relative maximum and one relative minimum; the leading coefficient is negative
   c. even-degree polynomial function with four relative maxima and three relative minima
   d. odd-degree polynomial function with three relative maxima and three relative minima; the left-most points are negative

4. Consider the function \( f(x) = x^4 - 8x^2 + 10 \). a. See margin.
   a. Evaluate \( f(x) \) for successive integers between -4 and 4 inclusive.
   b. Between what successive integers do the zeros appear? Approximate those zeros to the nearest tenth.
   c. State the ranges of \( x \) values where the values of \( f(x) \) are negative and ranges where the values of \( f(x) \) are positive. See margin.
   d. State the relative maximum(s) and relative minimum(s).
   e. Graph the function. See Solutions Manual.

Approximate the real zeros of each function to the nearest tenth.

5. \( f(x) = x^3 - x^2 + 1 \) 6. \( g(x) = x^4 + 3x^3 - 5 \)

6. \( -3.2, 1.1 \)

Graph each function. 7–10. See margin.

7. \( f(x) = x^3 \)
8. \( f(x) = x^3 - x^2 - 4x + 4 \)
9. \( f(x) = -3x^3 + 20x^2 - 36x + 16 \)
10. \( f(x) = x^4 - 7x^2 + x + 5 \)

State whether each graph is of odd degree or even degree. State the number of relative minima and relative maxima.

11. \( f(x) \) 12. \( f(x) \) 13. \( f(x) \)

14. Pharmacy A syringe is to deliver an injection of 2 cubic centimeters of medication. If the plunger is pulled out two centimeters to have the proper dosage, approximate the radius of the inside of the syringe to the nearest hundredth of a centimeter. Use the formula for the volume of a cylinder, \( V = \pi r^2 h \).

\[ 0.56 \text{ centimeters} \]
Approximate the real zeros of each function to the nearest tenth.

15. \( f(x) = x^3 - 2x^2 + 6 \) \(-1.3\)
16. \( h(x) = 2x^5 + 3x - 2 \) \(0.6\)
17. \( r(x) = x^5 - 6 \) \(1.4\)
18. \( g(x) = x^3 + 1 \) \(-1\)
19. \( f(x) = x^4 + 2x^2 - x^2 - 3 \)
20. \( p(x) = x^3 + 2x^2 - 3x - 5 \)
21. \( n(x) = 3x^3 - 16x^2 + 12x + 6 \)
22. \( h(x) = x^4 - 4x^2 + 2 \) \(-1.8, -0.8, 0.8, 1.8\)

Graph each function. 23–32. See Solutions Manual.

23. \( f(x) = 4x^6 \)
24. \( f(x) = 3x^5 \)
25. \( f(x) = x^2 - x \)
26. \( f(x) = -x^3 - 4x^2 \)
27. \( f(x) = x^3 + 5 \)
28. \( f(x) = x^4 - 81 \)
29. \( f(x) = 15x^3 - 16x^2 - x + 2 \)
30. \( f(x) = x^4 - 10x^2 + 9 \)
31. \( f(x) = -x^4 + x^3 + 8x^2 - 3 \)
32. \( f(x) = x^3 - x^2 - 8x + 12 \)

Approximate the real zeros of each function to the nearest tenth. Then use the functional values to graph the function.

33. \( r(x) = x^5 + 4x^4 - x^3 - 9x^2 + 3 \)
34. \( g(x) = x^4 - 9x^3 + 25x^2 - 2x + 6 \)
35. \( h(x) = x^3 - 3x^2 + 2 \)
36. \( f(x) = x^3 + 5x^2 - 9 \)
37. \( f(x) = x^4 + 7x + 1 \) \(-1.9, -0.1\)
38. \( p(x) = x^5 + x^4 - 2x^3 + 1 \) \(-2.0\)
39. \( q(x) = x^3 - x^2 - 8x + 12 \)
39b. The second graph is a vertical stretch of the first.

40. Find the relative maxima and relative minima of each function.
   a. \( f(x) = x^3 - 4x^2 + 8 \)
   b. \( f(x) = x^3 + 3x^2 - 12x \)
   \((0, 8), (2.67, -1.48)\) \((-3.24, 36.36), (1.24, -8.36)\)

41. Study the graphs for Exercises 23–32. Write a statement comparing the graphs of functions of even degree with those of functions of odd degree. See margin.

42. Geometry A function that represents the volume of a pyramid with a height of the same measure as the side of its square base is \( V(x) = \frac{1}{3}x^3 \).
   a. Graph the function. See margin.
   b. Find the zeros of the function. \( 0 \)
   c. Find the maximum and minimum of the function. There are none.
   d. Make a conjecture about how all of this data relates. See margin.

43. Aerospace Engineering The space shuttle has an external tank for the fuel that the main engines need for the launch. The tank is shaped like a capsule, a cylinder with a hemispherical dome at either end. The cylindrical part of the tank has a volume of 1170 cubic meters and a height of 17 meters more than the radius of the tank. What are the dimensions of the tank to the nearest tenth of a meter? (Hint: Use the formula for the volume of a cylinder.)

44. Physical Fitness An indoor running track is being built at a physical fitness center. It will consist of a rectangular region with a semicircle on each end. If the perimeter of the room is to be 200-meter running track, find the dimensions that will make the area of the rectangular region as large as possible.
Mixed Review

45. Business  The Energy Booster Company keeps their stock of Health Aid in a rectangular tank with sides that measure \((x - 1)\) cm, \((x + 3)\) cm, and \((x - 2)\) cm. Suppose they would like to bottle their Health Aid in \(x - 3\) containers of the same size. How many cubic centimeters of liquid will remain unbottled?  \(\text{(Lesson 8-2)}\)

46. Find the coordinates of the center and foci, and the lengths of the major and minor axes of the ellipse whose equation is \(\frac{x^2}{4} + \frac{y^2}{25} = 1\). \(\text{(Lesson 7-4)}\)

47. Name the coordinates of the vertex and the equation of the axis of symmetry for the graph of \(y = (x - 3)^2 - 11\). \(\text{(Lesson 6-6)}\)

48. Solve \(x^2 - 20x = -75\) by factoring. \(\text{(Lesson 6-2)}\) 15, 5

49. Solve \(-3y^2 = 18\). \(\text{(Lesson 5-9)}\) \(\pm \sqrt{6}\)

50. Solve the system of equations by using augmented matrices. \(\text{(Lesson 4-7)}\)

\[
\begin{align*}
2a + \frac{1}{3}b - 3c & = 19 \\
\frac{1}{2}a - b + 2c & = -16 \\
5a + 2b - 2c & = 50
\end{align*}
\]

4, 12, -3

51. Graph the line that passes through \((-2, 1)\) and is perpendicular to \(5 + 3x = -2y\). \(\text{(Lesson 2-3)}\) See margin.

52. Simplify \(\sqrt{64} \div \sqrt{4}\). \(\text{(Lesson 1-1)}\) 4

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Working on the Investigation

**Fill It Up!**

Select a coffee can that is a different size from the one you have been using. Repeat the experiment with the new coffee can using the same 50-mL increments. Record your measurements and data in a new chart like the one on page 475.

1. What is the capacity of the new coffee can?
2. How much water is in the can when the can is one-half full, one-fourth full, and three-fourths full? Make a note of these numbers.
3. Complete the entire chart. Were your calculations for the capacity of the can when it is one-half full, one-fourth full, and three-fourths full correct?

4. Make a drawing to illustrate the measurement scale of a dipstick for this coffee can. Describe the similarities and differences between the measurements of the two coffee can experiments. How are the scales alike and how are they different?

5. Use the information in your chart to make a table of ordered pairs \((x, y)\), where \(x\) represents the amount of water in the coffee can and \(y\) represents the percent of a full can. Graph the ordered pairs. Describe the graph.

6. Use the data from the first experiment to make another graph like the one described above. Describe the graph. What are the similarities or differences between the two experiments in terms of the graphs you have drawn?

Add the results of your work to your Investigation Folder.
8–3B Graphing Technology
Modeling Real-World Data

As we saw in Lesson 2-5B, you can use a graphing calculator to generate a scatter plot of data points and then determine the linear equation for the graph that best fits the plotted points. You can also use a graphing calculator to model data whose curve of best fit is a polynomial function.

Example

The table at the right shows how much time it takes each eight-hour work day to pay one day's worth of taxes. Draw a scatter plot and curve of best fit that shows how the year is related to hours worked.

First, convert hours in the table to minutes.

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of Minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1930</td>
<td>52</td>
</tr>
<tr>
<td>1940</td>
<td>89</td>
</tr>
<tr>
<td>1950</td>
<td>122</td>
</tr>
<tr>
<td>1960</td>
<td>140</td>
</tr>
<tr>
<td>1970</td>
<td>152</td>
</tr>
<tr>
<td>1980</td>
<td>160</td>
</tr>
<tr>
<td>1990</td>
<td>165</td>
</tr>
</tbody>
</table>

Next, set the window parameters. The values of the data suggest that you should use a viewing window [1920, 2000] by [0, 200] with a scale factor of 5 for the x-axis and 10 for the y-axis.

Then enter the data. Press STAT 1 to display lists for storing data. (If old data has been previously stored, clear the lists.) The years will be entered into list L1.

Enter: 1930 ENTER 1940 ENTER 1950 ENTER ... 1990 ENTER

Use the ▼ key to move the cursor to column L2 to enter the number of minutes worked to pay taxes.

Enter: 52 ENTER 89 ENTER 122 ENTER ... 165 ENTER

T E C H N O L O G Y

To clear a list, highlight the list heading and press CLEAR ENTER.
Now draw the scatter plot.

Enter: 2nd STAT 1 ENTER GRAPH

Next, compute and graph the equation of the curve of best fit. Try a cubic curve for this equation.

Enter: STAT 7 2nd L1 X=
2nd L2 ENTER Y=

VARS 5 7 GRAPH

The ZOOM IN feature allows you to move a cursor along the graph or scatter plot and read the coordinates of the points. Press ZOOM 2 and any of the arrow keys to observe what happens.

Approximately how many minutes should you expect to work each day to pay taxes in the year 2000?

In the year 2000, you should expect to work approximately 168.29 minutes each day to pay taxes.

**EXERCISES**

When an earthquake occurs, seismic waves are detected thousands of kilometers away from the epicenter within a matter of minutes. The table at the right gives the travel time of a primary seismic wave and the corresponding distance from the epicenter for several minutes.

<table>
<thead>
<tr>
<th>Travel Time (min)</th>
<th>Distance (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>400</td>
</tr>
<tr>
<td>2</td>
<td>800</td>
</tr>
<tr>
<td>5</td>
<td>3,200</td>
</tr>
<tr>
<td>7</td>
<td>6,250</td>
</tr>
<tr>
<td>10</td>
<td>10,000</td>
</tr>
<tr>
<td>12</td>
<td>14,447</td>
</tr>
<tr>
<td>13</td>
<td>20,000</td>
</tr>
</tbody>
</table>

1. Use a graphing calculator to draw a scatter plot for the data. State the viewing windows and scale factors that you used. See margin for scatter plot.

2. Calculate and graph curves of best fit that show how travel time is related to the distance. Try LinReg, QuadReg, CubicReg, and QuartReg. See margin.

3. Write the equation for the curve you think best fits the data. Describe the fit of the graph to the data. See students' work.

4. Based on the graph of a QuartReg curve, how far away from the epicenter will the wave be felt 8 1/2 minutes after the quake occurs? About how far are you from the epicenter if you feel the wave 15 minutes after the quake? about 4981 miles; about 14,447 miles

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