Mathematics in Ancient China

Chapter 7
Timeline
Early Timeline

- **Shang Dynasty:** Excavations near Huang River, dating to 1600 BC, showed “oracle bones” – tortoise shells with inscriptions used for divination. This is the source of what we know about early Chinese number systems.
Early Timeline
Han Dynasty (206 BC – 220 AD)

• System of Education especially for civil servants, i.e. scribes.
• Two important books:
  • *Zhou Bi Suan Jing* (Arithmetical Classic of the Gnomon and the Circular Paths of Heaven)
  • *Jiu Zhang Suan Shu* (Nine Chapters on the Mathematical Art)
Nine Chapters

- This second book, *Nine Chapters*, became central to mathematical work in China for centuries. It is by far the most important mathematical work of ancient China. Later scholars wrote commentaries on it in the same way that commentaries were written on *The Elements*. We’ll look at it in greater detail later.
Chapters in ... uh, the Nine Chapters

1. Field measurements, areas, fractions
2. Percentages and proportions
3. Distributions and proportions; arithmetic and geometric progressions
4. Land Measure; square and cube roots
5. Volumes of shapes useful for builders.
6. Fair distribution (taxes, grain, conscripts)
7. Excess and deficit problems
8. Matrix solutions
Song Dynasty (900 – 1279)

• Two Books by Zhu Shijie had topics such as:
  – Pascal’s triangle (350 years before Pascal)
  – Solution of simultaneous equations using matrix methods
  – “Celestial element method” of solving equations of higher degree.

• European algebra wouldn’t catch up to this level until the 1700’s.
Numeration

• Numerals on the Oracle Stones:
Numeration

1  2  3  4  5  6  7  8  9  10  11  12  13

20  30  40  50  80  88

100  162  200  500  600  656

1000  2000  3000  4000

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## Numeration

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Counting Rod System

1 2 3 4 5 6 7 8 9

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Counting Rods

• Counting rods allowed for a number of very quick calculations, including the basic four arithmetic operations, and extraction of roots.
Fractions

• From Nine Chapters:
• “If the denominator and numerator can be halved, halve them. If not, lay down the denominator and numerator, subtract the smaller number from the greater. Repeat the process to obtain the greatest common divisor (teng). Simplify the original fraction by dividing both numbers by the teng.”
Fractions

• Addition and subtraction were done as we do them but without finding least common denominators – the common denominator is just the product of the two denominators. The fraction is simplified after adding or subtracting.
Fractions

- Multiplication was done as we do it.
- Division was done by first getting common denominators, then inverting and multiplying so that the common denominators cancel. Then the fraction was simplified.
Negative numbers?

- **Red** and black rods, or rods laid diagonally over others.
- “For subtractions – with the same signs, take away one from the other; with different signs, add one to the other; positive taken from nothing makes negative, negative from nothing makes positive.”
- “For addition – with different signs subtract one from the other; with the same signs add one to the other; positive and nothing makes positive; negative and nothing makes negative.”
Approximations of $\pi$

- Liu Hui, 260 AD: 3.1416 (by inscribing hexagon in circle, using the Pythagorean Theorem to approximate successively polygons of sides 12, 24, ..., 96).
- Zu Chongzhi, 480 AD: between 3.1415926 and 3.1415927 (by similar method, but moving past 96 to oh, say 24,576).
Magic Squares
Lo Shu

- The semi-mythical Emperor Yu, (circa 2197 BC) walking along the banks of the Luo River, looked down to see the Divine Turtle. On the back of his shell was a strange design.
Lo Shu

• When the design on the back was translated into numbers, it gave the 3x3 magic square.

• Saying “the” 3x3 magic square is appropriate because it is unique up to rotations and reflections.
He Tu

- According to legend, the *He Tu* is said to have appeared to Emperor Yu on the back of (or from the hoof-prints of) a Dragon-Horse springing out of the Huang (Yellow) River.
He Tu

• When it was translated into numbers, it gave a cross-shaped array.
• To understand its meaning is to understand the structure of the universe, apparently.
• Or, at least to understand that, disregarding the central 5, the odds and evens both add to 20.
Magic Squares

• Yang Hui, “Continuation of Ancient Mathematical Methods for Elucidating the Strange Properties of Numbers”, 1275.
Order 3

- Arrange 1-9 in three rows slanting downward to the right.

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Order 3

• Arrange 1-9 in three rows slanting downward to the right.

• Exchange the head (1) and the shoe (9).
Order 3

• Arrange 1-9 in three rows slanting downward to the right.

• Exchange the head (1) and the shoe (9).

• Exchange the 7 and 3.
Order 3

- Arrange 1-9 in three rows slanting downward to the right.
- Exchange the head (1) and the shoe (9).
- Exchange the 7 and 3.
- Lower 9, and raise 1.
Order 3

• Arrange 1-9 in three rows slanting downward to the right.

• Exchange the head (1) and the shoe (9).

• Exchange the 7 and 3.

• Lower 9, and raise 1.

• Skootch* in the 3 and 7

*technical term
Order 3 – The Lo Shu

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Order 4

- Write 1 – 16 in four rows.

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Order 4

- Write 1 – 16 in four rows.
- Exchange corners of outer square

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Order 4

• Write 1 – 16 in four rows.

• Exchange corners of outer square

• Exchange the corners of inner square.

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Order 4

• Write 1 – 16 in four rows.

• Exchange corners of outer square

     16  2  3  13
     5  11  10  8

• Exchange the corners of inner square.

     9  7  6  12
     4  14  15  1
Order 4

• Write 1 – 16 in four rows.
• Exchange corners of outer square
• Exchange the corners of inner square.
• Voila! Sum is 34.
Order 4

• Other magic squares of order 4 are possible for different initial arrangements of the numbers 1 – 16.

\[
\begin{array}{cccc}
13 & 9 & 5 & 1 \\
14 & 10 & 6 & 2 \\
15 & 11 & 7 & 3 \\
16 & 12 & 8 & 4 \\
\end{array}
\]

\[
\begin{array}{cccc}
4 & 9 & 5 & 16 \\
14 & 7 & 11 & 2 \\
15 & 6 & 10 & 3 \\
1 & 12 & 8 & 13 \\
\end{array}
\]
Order 5, 6, 7, ....

• Yang Hui constructed magic squares of orders up through 10, although some were incomplete.
A Little About Magic Squares

• *Normal* magic squares of order $n$ are $n \times n$ arrays containing each number from 1 through $n^2$. They exist for all $n > 2$.

• The sum of each row, column, and diagonal is the *magic number* $M$ which for normal magic squares depends only on $n$.

• $M = \frac{n(n^2 + 1)}{2}$. For the first few $n$’s this is 15, 34, 65, 111, 175 . . .

• For $n$ odd, the number in the central cell is $\frac{M}{n}$.