The Birth of Hyperbolic Geometry
Carl Friederich Gauss

• There is some evidence to suggest that Gauss began studying the problem of the fifth postulate as early as 1789, when he was 12. We know in letters that he had done substantial work of the course of many years:
Carl Friederich Gauss

• “On the supposition that Euclidean geometry is not valid, it is easy to show that similar figures do not exist; in that case, the angles of an equilateral triangle vary with the side in which I see no absurdity at all. The angle is a function of the side and the sides are functions of the angle, a function which, of course, at the same time involves a constant length. It seems somewhat of a paradox to say that a constant length could be given a priori as it were, but in this again I see nothing inconsistent. Indeed it would be desirable that Euclidean geometry were not valid, for then we should possess a general a priori standard of measure.” – Letter to Gerling, 1816
Carl Friederich Gauss

• "I am convinced more and more that the necessary truth of our geometry cannot be demonstrated, at least not by the human intellect to the human understanding. Perhaps in another world, we may gain other insights into the nature of space which at present are unattainable to us. Until then we must consider geometry as of equal rank not with arithmetic, which is purely a priori, but with mechanics.‘‘ – Letter to Olbers, 1817
Carl Friederich Gauss

- "There is no doubt that it can be rigorously established that the sum of the angles of a rectilinear triangle cannot exceed 180°. But it is otherwise with the statement that the sum of the angles cannot be less than 180°; this is the real Gordian knot, the rocks which cause the wreck of all.... I have been occupied with the problem over thirty years and I doubt if anyone has given it more serious attention, though I have never published anything concerning it."

- ("Over thirty years" puts the start date before 1794, or before age 17.)
Carl Friederich Gauss

"The assumption that the angle sum is less than 180° leads to a peculiar geometry, entirely different from Euclidean, but throughout consistent with itself. I have developed this geometry to my own satisfaction so that I can solve every problem that arises in it with the exception of the determination of a certain constant which cannot be determined a priori. The larger one assumes this constant the more nearly one approaches the Euclidean geometry, an infinitely large value makes the two coincide. The theorems of this geometry seem in part paradoxical, and to the unpracticed absurd; but on a closer and calm reflection it is found that in themselves they contain nothing impossible...."
Carl Friederich Gauss

• “All my efforts to discover some contradiction, some inconsistency in this Non-Euclidean geometry have been fruitless, the one thing in it that seems contrary to reason is that space would have to contain a definitely determinate (though to us unknown) linear magnitude. However, it seems to me that notwithstanding the meaningless word-wisdom of the metaphysicians we know really too little, or nothing, concerning the true nature of space to confound what appears unnatural with the absolutely impossible. Should Non-Euclidean geometry be true, and this constant bear some relation to magnitudes which come within the domain of terrestrial or celestial measurement, it could be determined a posteriori.” – Letter to F. A. Taurinus, 1824.
Carl Friederich Gauss

• “There is also another subject, which with me is nearly forty years old, to which I have again given some thought during leisure hours, I mean the foundations of geometry.... Here, too, I have consolidated many things, and my convictions has, if possible become more firm that geometry cannot be completely established on a priori grounds. In the mean time I shall probably not for a long time yet put my very extended investigations concerning this matter in shape for publication, possibly not while I live, for I fear the cry of the Bœotians which would arise should I express my whole view on this matter.” - Letter to Bessel, 1829.

• Forty year earlier would have been about 1789, with Gauss about 12 years of age.
Carl Friederich Gauss

- The “Bœotians” he refers to are the followers of the philosopher Immanuel Kant, who insisted that “the concept of [Euclidean] space is by no means of empirical origin, but is an inevitable necessity of thought.”
- Gauss never did make his work on non-Euclidean geometry known publicly, partly because he didn’t want to be drawn into debates with the Kantians, and partly because he was a perfectionist and only published completed works of his mathematical art. “Few but ripe.”
János Bolyai

- 1802 – 1860, Hungary
- Son of Farkas Bolyai
- Mastered calculus at age 13
- Attended military academy, graduating in 1823 (completing a 7-year course in 4 years).
János Bolyai

- Neither smoked nor drank (not even coffee)
- Spoke nine languages, including Chinese and Tibetan.
- “The best swordsman and dancer in the Austro-Hungarian Imperial Army.”
- Fought and won 13 successive duels (with swords), playing the violin after every two victories.
- Thought to be of somewhat fiery temperament.
- Duh.
János Bolyai

- Began his discoveries in geometry in the early 1820’s.
- Plagued with intermittent fevers, he retired from the army in 1833 as a semi-invalid, and lived on a pension.
- Met Rozália Kibédi Orbán and they lived together at Domáld from 1834 (He was too poor to marry). They had two children.

- Married in 1849, separated in 1852.
- Died of pneumonia in 1860, at the age of 57.
- Died without gaining any public recognition for his geometrical contributions.
János Bolyai

• By the way, he didn’t look like the picture on the right, which is from a postage stamp honoring his life.
• We don’t really have any reliable portraits. However. . . .
János Bolyai
János Bolyai

• Dénes Boylai

• [http://www.titoktan.hu/Bolyai_a.htm](http://www.titoktan.hu/Bolyai_a.htm)

• György Klapka
János Bolyai
On 3 November 1823 he wrote to his father:

“I am resolved to publish a work on parallels as soon as I can complete and arrange the material, and the opportunity arises. At the moment I still do not clearly see my way through, but the path which I have followed is almost certain to lead me to my goal, provided it is at all possible. I have not quite reached it, but I have discovered things so wonderful that I was astonished and it would be an everlasting pity if these things were lost. When you, my dear father, see them, you will understand. All I can say at present is that out of nothing I have created a strange new world. All that I have sent you previously is like a house of cards in comparison with a tower.”
János Bolyai

• In an 1825 letter to János, Farkas advised:
  “It seems to me advisable, if you have actually succeeded in obtaining a solution of the problem, that, for a two-fold reason its publication be hastened: first, because ideas easily pass from one to another who, in that case, can publish them; secondly, because it seems to be true that many things have, as it were, an epoch in which they are discovered in several places simultaneously, much as the violets appear on all sides in the springtime.

As it turns out, this was somewhat prophetic.
János Bolyai

• János published about 25 pages on his ideas as an appendix to his father’s *Tentamen*, in 1831. As it happens, that is all he ever published.

• Farkas sent a copy to his friend Carl Friederich Gauss, under the assumption that Gauss would be impressed by his son’s work.
János Bolyai

- In a letter to Farkas in 1832, Gauss wrote:
  “If I begin by saying that I dare not praise this work, you will of course be surprised for a moment; but I cannot do otherwise. To praise it would amount to praising myself. For the entire content of the work, the approach which your son has taken, and the results to which he is led, coincide almost exactly with my own meditations and which have occupied by mind for the past thirty or thirty five years.... It was my plan to put in all down on paper eventually, so that at least it would not perish with me. So I am greatly surprised to be spared the effort, and am overjoyed that it happens to be the son of my old friend who outstrips me in such a remarkable way.”
János Bolyai

• In another letter to a friend, Gauss wrote:
  “Let me add further that I have this day received from Hungary a little work on non-Euclidean geometry in which I find all my own ideas and results developed with great elegance, although in a form so concise as to offer great difficulty to anyone not familiar with the subject. I regard this young geometer Bolyai as a genius of first order.”
János Bolyai

• Despite Gauss’ praise of his work, János was extremely disappointed and never published any more of the work in the 14,000 pages of notes he left.

• He was even more disappointed to discover that Gauss was not the only one who had “discovered” these ideas.
Nicolai Ivanovich Lobachevsky

• Nicolai Ivanovich Lobachevsky was the son of Ivan Maksimovich Lobachevsky and Praskovia Alexandrovna Lobachevskaya.

• Sorry, I was channeling the Brothers Karamazov for a minute there.
Nicolai Ivanovich Lobachevsky

- Born in 1792 or 1793 in Nizhny Novgorod (Gorky), about 250 miles SE of Moscow.
- Moved to Kazan, on the western edge of Siberia, with his widowed mother in 1800 (climate: think Montreal, Green Bay, Duluth, Fargo).
- Attended Kazan Gymnasium on government scholarship and entered Kazan University in 1807.
Nicolai Ivanovich Lobachevsky

- Graduated from Kazan University in 1811.
- Began lecturing in mathematics, physics, and astronomy at Kazan in 1814, and was promoted to “extraordinary” professor in 1816, and full professor in 1822, at the age of 29.
- Married Lady Varvara Alexejevna Moisieva in 1832, many children.
- “Retired” from Kazan in 1846, having had a successful career as a university administrator.
- His health declined after retirement, and he was left in poverty.
- Died in 1856 in Kazan.
Nicolai Ivanovich Lobachevsky

• Studied mathematics under Johann Bartell, once a teacher of Gauss.
• As late at 1823 was still attempting to “prove” Euclid’s V, though he recognized “they do not deserve to be called proofs.”
• His memoir *On the Foundation of Geometry*, was published in installments in the Kazan Messenger in 1829-1830
• This was the first account of non-Euclidean geometry to be published.
• (Remember that Bolyai’s father encouraged him to publish in 1825, and Bolyai’s work finally appeared in 1831.)
• Submitted *On the Foundations of Geometry* to the St. Petersburg Academy of Sciences (rejected because two integrals were questioned).
Nicolai Ivanovich Lobachevsky

- Several publications of Lobachevsky’s ideas followed, in Moscow (1835), France (1837), and Germany (1840).
- Of the German publication, which Nicolai sent to him, Gauss said:
  
  “I have recently had occasion to look through again that little volume by Lobachevsky. . . I have found in Lobachevsky’s work nothing that is new to me, but the development is made in a way different from that which I have followed, and certainly by Lobachevsky in a skillful way and a truly geometrical spirit.”
Nicolai Ivanovich Lobachevsky

• Gauss arranged for Lobachevsky to be a corresponding member of the Gottingen Academy of Sciences, but gave no public support of the ideas.

• Why not Bolyai? Who knows?

• Like Bolyai, Lobachevsky died without any real recognition for their geometrical contributions.

• Although they may not have strictly been the first to “discover” it, they were the first to believe in its non-contradictory existence, and have guts enough to say so.
AD is the perpendicular from A to BC. AE is perpendicular to AD. Within the angle EAD, some lines (such as AF) will meet BC. Assume that AE is not the only line which does not meet BC, so let AG be another such line. AF is a cutting line and AG is a non-cutting line. There must be a boundary between cutting and non-cutting lines and we may take AH as this boundary.