Animals’ Number Sense

A chimpanzee named Sheba was trained by Sarah Boysen to understand the meanings of the digits 1 – 9. Sheba was able to perform arithmetic with them. In one experiment, a number of oranges were hidden in various places in Sheba’s cage – for example, two oranges under a chair, and 4 more in a box. Sheba’s task was to explore her cage, then come back and choose the digit that matched the total number of oranges. She succeeded from the very first trial. Then, rather than oranges, the experimenters hid the digit “2” and the digit “4.” Sheba explored the hiding places and then chose the digit “6.”

Two macaques named Able and Baker were able to distinguish the largest among up to five digits. When presented with a string of digits they would use a joystick to point to one of them. They then received that number of fruit candies. They soon learned to pick the largest number. For example, with the sequence “5 2 1 8 3” they would pick 8 – not infallibly, but much more often than chance alone would predict.

Infants’ Number Sense

Infants can discriminate between 2 and 3 objects a few days after birth. When infants are repeatedly shown cards with two objects on them, eventually they get bored and start spending less and less time looking at them. When a card with 3 objects is then shown, the infant spends more time staring at it, demonstrating that it can distinguish between 2 and 3 objects. These are called habituation experiments because they depend on the infant becoming habituated to certain stimuli and then acting surprised (indicated by staring longer) at different stimuli.

Infants can also distinguish between words of two and three syllables a few days after birth.

Four and five month old infants express surprise at “impossible” arithmetic, such as when two puppets are put behind a screen and the screen drops to reveal either one or three puppets instead. Thus it seems babies know 1+1 = 2, and in similar experiments it is show that they also know that 2 – 1 = 1.

Slightly older infants know that 2 + 1 = 3 and 3 – 1 = 2.

Babies as young as 6 months can discriminate between 2 and 3 actions, such as a puppet jumping 2 or 3 times.

In one experiment using six- through eight-month old babies, babies were presented two stimuli: a card with either two or three common objects on it, and a recording of a drum beating either two or three times. Babies spent more time looking at two-object cards when hearing two beats, and three-object cards when hearing three beats. Thus it seems they could coordinate the numerosity of objects in space and sounds over time.

## Math Ed 305 – Summary Sheet on Counting Principles – Part I

<table>
<thead>
<tr>
<th>Gallistel &amp; Gelman’s Principles</th>
<th>Refinements (From Fuson, Steffe &amp; Cobb; Dahaene)</th>
<th>What Can Go Wrong?</th>
</tr>
</thead>
</table>
| **One-One Principle:** The items being counted get matched with number words in such a way that one and only one number word is used for each item in the array. | Part of establishing the matching between number words and items is matching each item with an indicating act (pointing, moving, resting your eyes on an object) in a one-one way. | (a) Performing an indicating act not attached to an item (e.g. pointing to a blank space)  
(b) Indicating two or more items with one act (e.g. pointing between two items and having it count for both).  
(c) Skipping an item (e.g. not pointing to some item)  
(d) Indicating at item twice (e.g returning to point to an item again) |
| | The other part of establishing the matching between number words and items is matching each indicating act with a counting word in a one-one way | Although (b) and (c) may look the same, they are subtly different.  
Avoiding (c) and (d) requires a way of remembering what has been counted; often this is done by partitioning the items into two groups – the counted and the uncounted. It can be done by moving items, lining them up and going from left to right, etc. |
| | Coordinating the above two actions | (a) Saying a counting word without pointing.  
(b) Saying two or more number words while pointing only once  
(c) Pointing without saying a counting number  
(d) Pointing twice or more while saying one counting number. |
| | | Although (a) and (b) may look the same, they are subtly different.  
Typically, students have less trouble with coordinating number words and indicating acts than they do coordinating items and indicating acts. |
<p>| | | These are coordinated by using the same indicating acts, and by keeping track to make sure all the objects are used. |</p>
<table>
<thead>
<tr>
<th>How-to-Count Principles</th>
<th>Gallistel &amp; Gelman’s Principles</th>
<th>Refinements (From Fuson, Steffe &amp; Cobb; Dahaene)</th>
<th>What Can Go Wrong?</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Stable Order Principle: Number words being matched with objects must be said in the correct order.</td>
<td>Reciting the counting number sequence may not be stable for young children – they may repeat words, skip others, get still others out of order. The good news is that most children don’t have difficulty with this by the time they enter elementary school.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The Cardinal Principle: The last counting word said in the counting process names the cardinality of the set.</td>
<td>Some researchers believe this is learned by counting small groups of items and recognizing that the last number in the sequence is the same as what they get by subitizing. Others believe it is learned as a rule or convention.</td>
<td>Most children learn this without difficulty.</td>
<td></td>
</tr>
<tr>
<td>The Abstraction Principle: Counting can be performed on any collection of objects or events, even abstract ones.</td>
<td>Steffe and Cobb believe that children progress from being able to count (a) physical objects in plain sight, to (b) hidden physical objects, to (c) physical actions such as pointing, tapping, or raising a finger, to (d) verbalizations (such as number words themselves), to (e) whatever they can imagine.</td>
<td>If children do not learn how to count abstract entities, like counting words, they cannot “double count” which forms the basis for solving a wide variety of problems.</td>
<td></td>
</tr>
<tr>
<td>The Order-Irrelevance Principle: The order in which objects are matched with numbers is irrelevant.</td>
<td>Understanding this could be the basis for some understandings about arithmetic, e.g. commutativity and associativity.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
For counting to be correct, (a) each indicating act must be directed toward an object, (b) each indicating act must not be directed toward more than one object, (c) every object must be indicated, and (d) no object can be indicated more than once. Satisfying (c) and (d) requires a way of remembering what has been counted.

Keeping track by (a) “marking” individual counted objects so that it is clear that the object has been counted (moving objects from an uncounted to a counted pile; pointing in which each object is physically or mentally “marked” as the object is counted) and (b) using a linear ordering on the objects.

**Indicating acts:** (a) moving objects from a pile of uncounted objects to a pile of counted objects or (b) some variation of pointing at unmoved objects, including the later internalization of pointing as eye fixation.

For counting to be correct, (a) one word must correspond to one indicating act, (b) one indicating act must correspond to one word and (c) the words must come from the standard number-word sequence.